1

(a) Acceleration is directly proportional to the displacement and it is always in the opposite direction to its displacement (or directed to the equilibrium position)

(b) **(i)** $0.034\ m$

(ii)
$$\omega = \frac{v_0}{x_0}$$
$$= \frac{0.60}{0.034}$$
$$= 17.6\ rad\ s^{-1}$$

(c) **(i)** Upward taken to be positive,
$F_{net} = N - W = -mx\omega^2$
When object loses contact, $N = 0$

$$mg = mx\omega^2$$
$$x = \frac{g}{\omega^2}$$
$$= \frac{9.81}{17.647^2}$$
$$= 0.0315\ m$$

(ii) C marked at $(0.032, 0.2)$

2

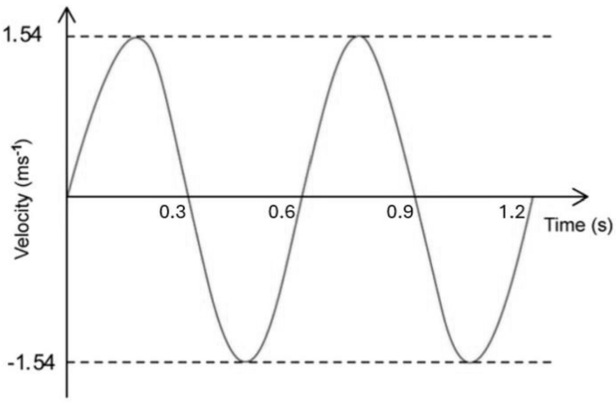
(a) **(i)** The mass passes through the equilibrium position P twice in a cycle, meaning that there are 100 complete cycles per minute.

period, $T = \frac{1}{f} = \frac{60}{100} = 0.600\ s$

(ii) At equilibrium position $x = 0$, the kinetic energy of the mass E_k is at its maximum

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.600} = 10.47\ rad\ s^{-1}$$
$$E_K = \frac{1}{2}m\omega^2x_0^2 = \frac{1}{2}(0.42)(10.47)^2x_0^2$$
$$x_0 = \sqrt{\frac{2(0.500)}{0.42(10.47)^2}} = 0.147\ m \approx 15cm$$

(iii)



(b) **(i)** No change

(ii) Frequency increases by a factor of $\sqrt{2}$

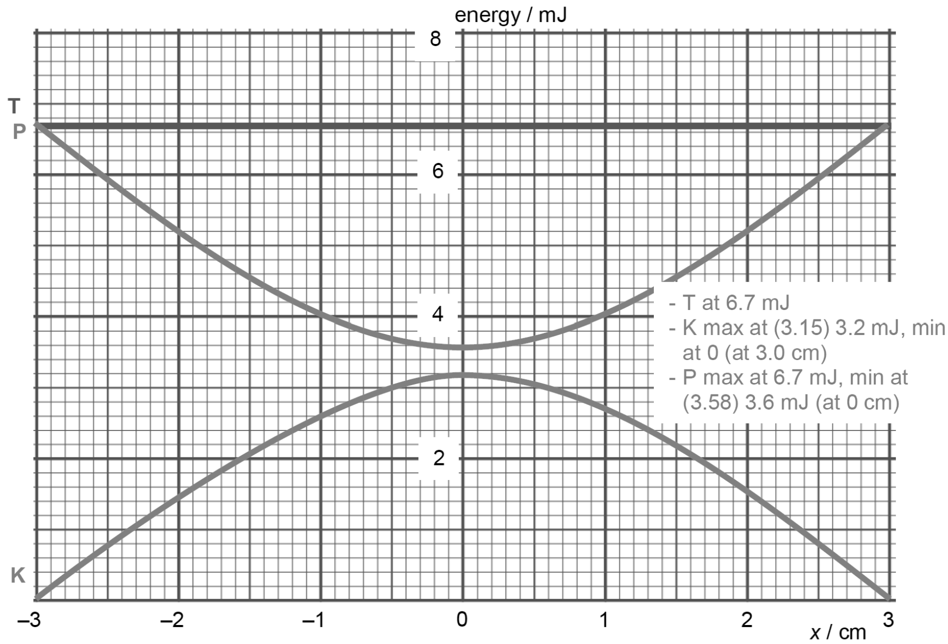
3

(a) **(i)** $x_A = 6.2\ cm$
$x_B = 0.2\ cm$

(ii) Total energy of the system = maximum PE at the amplitude position

$$= \frac{1}{2}k_A x_A^2 + \frac{1}{2}k_B x_B^2$$
$$= \frac{1}{2}(3.5)(0.062)^2 + \frac{1}{2}(3.5)(0.002)^2$$
$$= 6.73 \times 10^{-3}\ J$$

(b) **(i)**
(ii)
(iii)



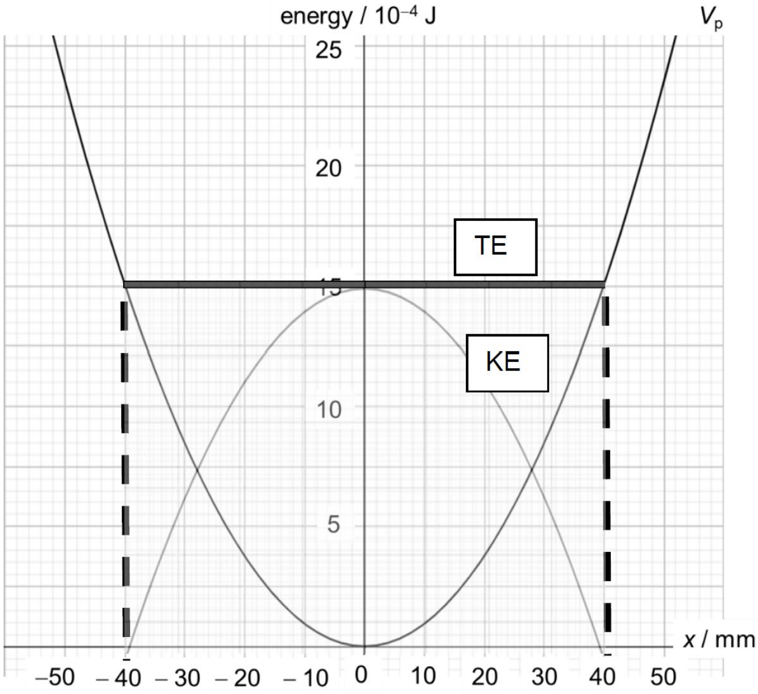
(c) The maximum energy of the oscillation remains constant, hence by energy conservation, the maximum KE at equilibrium position remains unchanged. However, with a larger mass, the maximum speed is therefore reduced

4

(a) Its acceleration is directly proportional to its displacement from a fixed point (equilibrium position) and is always directed towards that fixed point.

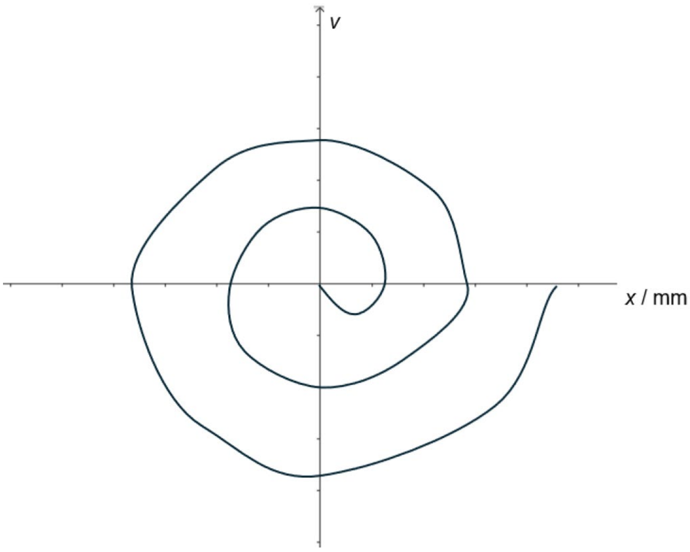
(b) Gain in GPE = mgh
$(0.150)(9.81)(1.0 \times 10^{-3}) = 1.4715 \times 10^{-3}$
$\approx 1.47 \times 10^{-3}\ J$

(c) **(i)**
(ii)



(d) From calculation in (b), total energy is $1.5 \times 10^{-3}\ J$
So the horizontal displacement should be $40\ mm$

(e)



5

(a) **(i)** reduction in energy / amplitude (of the oscillations)
due to force opposing motion / resistive forces / dissipative forces

(ii) amplitude is decreasing (very) gradually / oscillations would continue for a long time / many complete oscillations, hence light damping

(b) **(i)** Frequency $= \frac{1}{0.3} = 3.3\ Hz$

(ii) $E_T = \dfrac{1}{2}mv_0^2 = \dfrac{1}{2}m\omega^2 x_0^2$

$$= \frac{1}{2}(0.065)\left(\frac{2\pi}{0.3}\right)^2 (1.5 \times 10^{-2})^2$$
$$= 3.21\ mJ$$

(c) amplitude reduces exponentially / amplitude decreases by a smaller amount (for each oscillation) / does not decrease linearly / decrease at a decreasing rate, so amplitude will not be 0.7 cm.

(d) pointer in ammeter or car suspension system (critical damping)

6

(a) At equilibrium position, $F_{net} = 0$

$kx_0 = mg$

After displacement of x downwards,
$ma = k(x + x_0) - mg$
$= k(x + x_0) - kx_0$
$= kx$

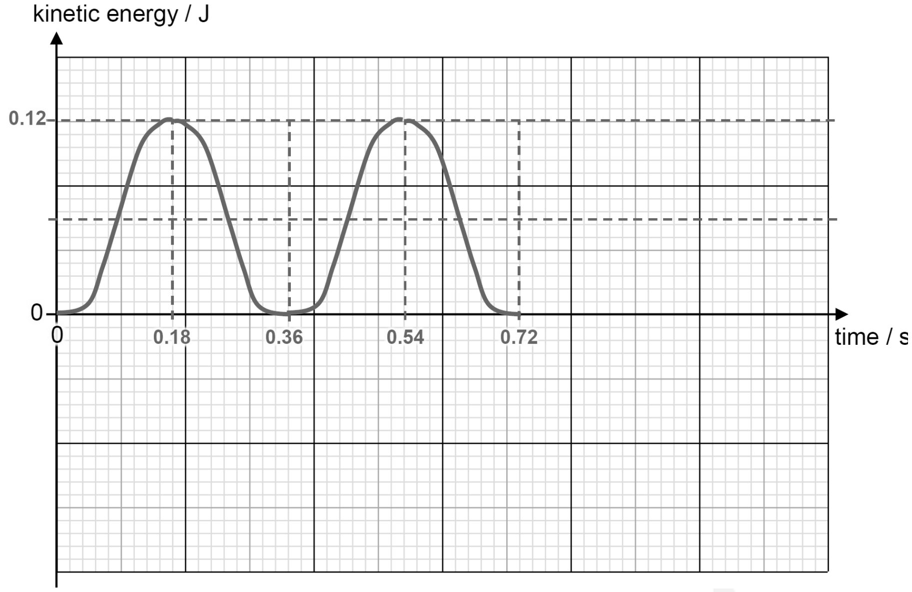
Since a and x are opposite in directions,
$ma = -kx$
$a = -\frac{k}{m}x$

(b) $a = -\omega^2 x$
$\omega^2 = \frac{k}{m}$
$\left(\frac{2\pi}{T}\right)^2 = \frac{k}{m}$
$T = 2\pi\sqrt{\frac{m}{k}}$

(c) $T = \frac{t}{10}$
$T = \frac{7.2}{10} = 0.72\ s$
$\frac{\Delta T}{T} = \frac{\Delta t}{t} = \frac{0.2}{7.2}$
$k = \frac{4\pi^2 m}{T^2} = \frac{4\pi^2(0.120)}{(0.72)^2} = 9.1385$
$\frac{\Delta k}{k} = \frac{\Delta m}{m} + \frac{2\Delta T}{T} = \frac{1}{100} + 2\left(\frac{0.2}{7.2}\right) = 0.065556$
$\Delta k = 0.065556(9.1385) = 0.6$
$(k \pm \Delta k) = (9.1 \pm 0.6)\ N\ m^{-1}$

(d) Total energy = maximum E_k
$= \frac{1}{2}m\omega^2 x_0^2$
$= \frac{1}{2}(0.120)\left(\frac{2\pi}{0.72}\right)^2 (0.16)^2$
$= 0.117\ J$

(e)



7

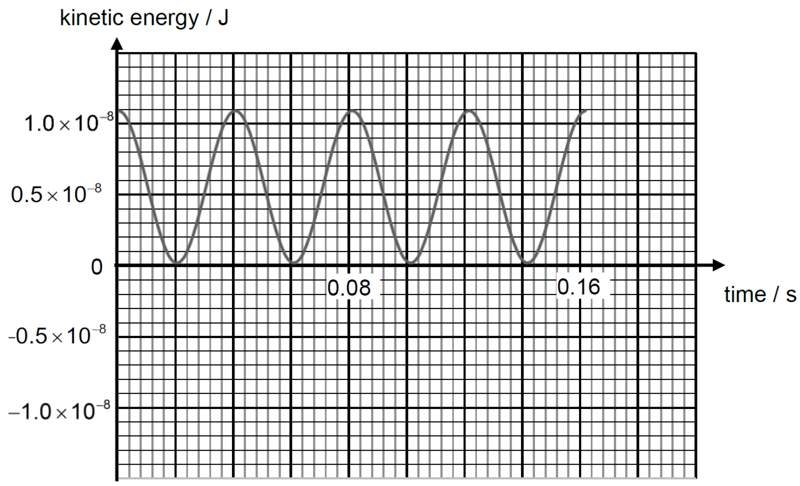
(b) **(i)** $\omega = 2\pi f = 2\pi(33) = 66\pi$
$x = 2.1 \times 10^{-3} \sin(66\pi t)$

(ii) $x = 2.1 \times 10^{-3} \sin(66\pi t)$
$1.3 \times 10^{-3} = 2.1 \times 10^{-3} \sin(66\pi t)$
$t = 3.2 \times 10^{-3}\ s$
$v = (2.1 \times 10^{-3})(66\pi)\cos(66\pi t)$
$= (2.1 \times 10^{-3})(66\pi)\cos\left(66\pi(3.2 \times 10^{-3})\right)$
$= 0.34\ m\ s^{-1}$

(c) $v = 9.2 \times 10^{-2} \cos(77t)$
$v_0 = \omega x_0 = 9.2 \times 10^{-2}$
$77x_0 = 9.2 \times 10^{-2}$
$x_0 = 1.2 \times 10^{-3}\ m$

(d) **(i)** $v = 9.2 \times 10^{-2} \cos(77t)$
Max KE $= \frac{1}{2}mv_0^2 = \frac{1}{2}(2.5 \times 10^{-6})(9.2 \times 10^{-2})^2 = 1.1 \times 10^{-8}\ J$

(ii)



(iii) Frequency, $f = \frac{77}{2\pi}\ Hz$
Time interval is $\frac{3}{4}T = \frac{3}{4f} = \frac{3(2\pi)}{4(77)} = 0.061\ s$

(e) **(i) 1.** A forced oscillation is one which is driven by an external force such that energy is supplied to the oscillation.

(i) 2. The effect illustrated is called resonance.

(ii) same starting point and lower graph peak
maximum amplitude at same / lower frequency within original shape

8

(a) **(i)** $v = f\lambda$
$0.90 = f(0.30)$
$f = 3.0\ Hz$

(ii)
$$f = \frac{1}{2\pi}\sqrt{\frac{28}{m}}$$
$m = 0.0788\ kg$

(iii) 1. Driving force is larger due to the larger amplitude of wave, hence amplitude of the vertical oscillations will increase.

(iii) 2. Wavelength of wave increases. Since the speed remains the same, by $v = f\lambda$, driving frequency of wave will decrease. Since driving frequency is not equal to natural frequency, amplitude will decrease as the system is no longer in resonance

(iii) 3. Damping force increases and hence, amplitude decreases.

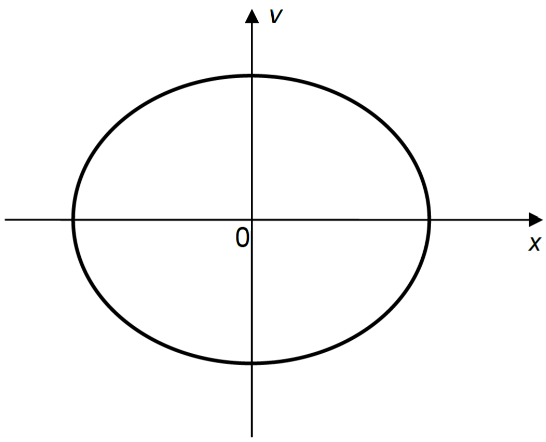
9

(b) **(i)** g and r are constant so a is proportional to x
negative sign shows a and x are in opposite directions

(ii)
$$\omega^2 = \frac{g}{r}$$
$$\left(\frac{2\pi}{T}\right)^2 = \frac{9.81}{0.28}$$
$T = 1.06\ s$

(iii) $x = x_0 \sin \omega t$
$$1.5 = 2.0 \sin\left(\frac{2\pi}{1.1}t\right)$$
$t = 0.143\ s$

(iv)



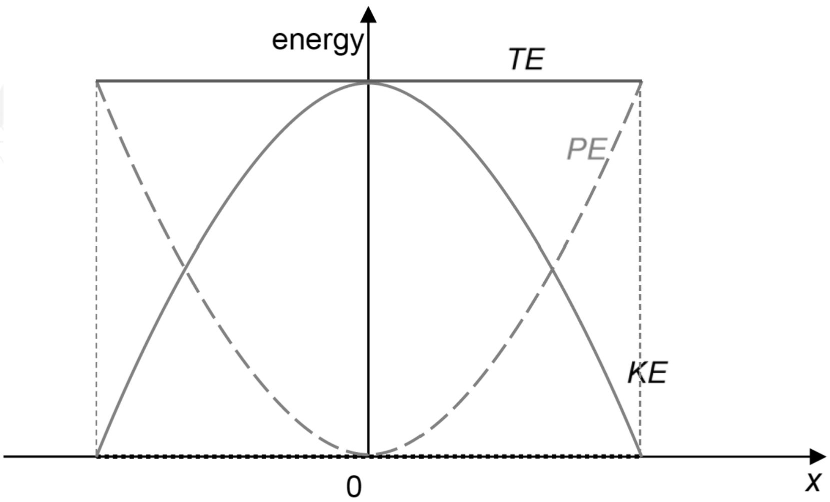
$$v_0 = \omega x_0$$
$$v_0 = \frac{2\pi}{T} x_0$$
$$v_0 = \frac{2\pi}{1.1}(0.020) = 0.118\ m\ s^{-1}$$

(v) Total energy = max KE

$= \frac{1}{2}(0.100)(1.1)^2$

$= 0.61\ mJ$

(vi)



(vii)
$$N_{28}T_{28} = N_{29}T_{29}$$
$$N\sqrt{28} = (N-1)(\sqrt{29})$$
$$N = 57.5$$
$$t = 57.5\left(2\pi\sqrt{\frac{0.29}{9.81}}\right)$$
$$= 62\ s$$

10

(a) **(i)** $loss\ = mgh = 0.400 \times 9.81 \times 0.150 = 0.589\ J$

(ii) Gain in EPE = $\frac{1}{2}Fx = \frac{1}{2}mgx = \frac{1}{2}(0.400)(9.81)(0.150) = 0.294\ J$

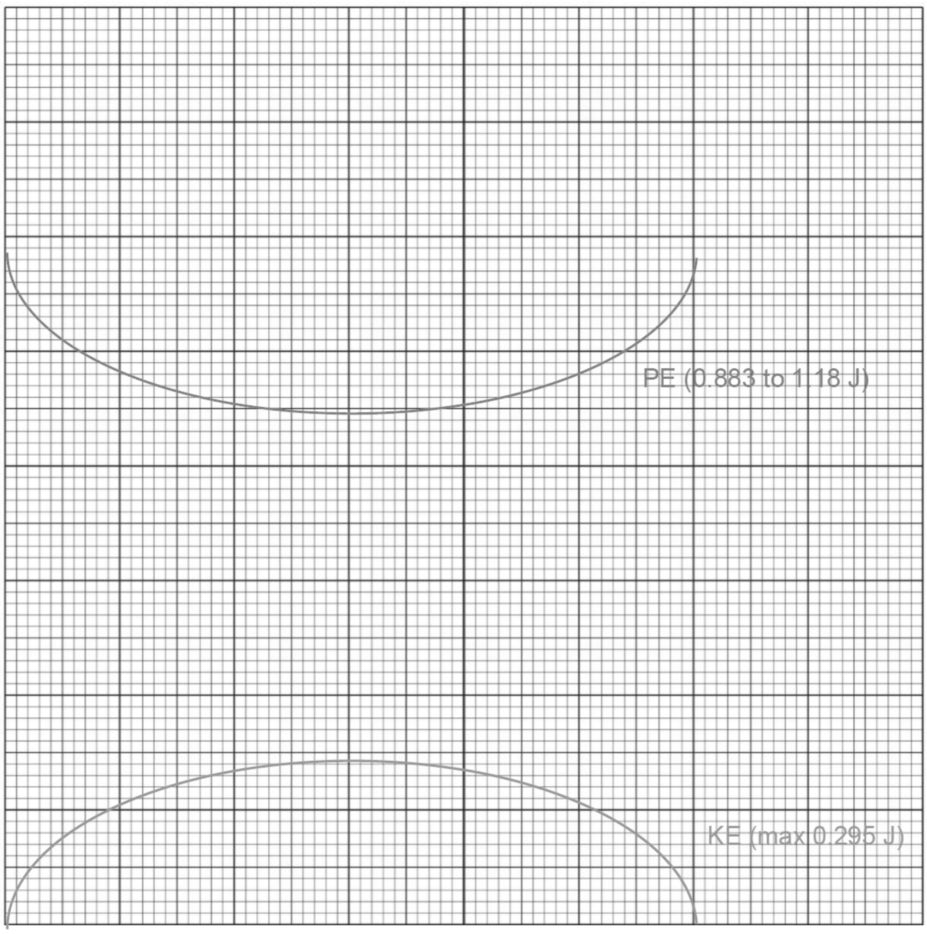
(b) When the mass is lowered gently, there is a resistive or supporting force that prevents the mass from accelerating. As a result, part of the loss in gravitational potential energy is into the work done against the supporting force and the gain in elastic potential energy is less than the loss in gravitational potential energy.

(c) Spring constant = $\frac{F}{e} = \frac{mg}{e} = \frac{0.400(9.81)}{0.15} = 26.16$

Net force
$= T - W$
$= ke - mg$
$= 26.16(2 \times 0.15) - 0.400(9.81)$
$= 3.924$

Since $F = -m\omega^2 x$
At amplitude position, $3.924 = m\omega^2 x_0 = 0.400\omega^2(0.15)$
$\omega = 8.09\ rad\ s^{-1}$

(d)



KE graph (correct shape and values with reference to equilibrium)
GPE graph (correct shape and relative values at amplitude and equilibrium)
GPE graph (correct absolute values)