1

(a) **(i)** Gravitational force provides for the centripetal force

$$\frac{GMm}{R^2} = \frac{mv^2}{R} \ (v \ is \ the \ tangential \ speed)$$
$$\frac{1}{2}mv^2 = \frac{GMm}{2R}$$

Hence, $E_K = \frac{GMm}{2R}$

(ii) Total energy = sum of GPE and KE

$$E_T = -\frac{GMm}{R} + \frac{GMm}{2r}$$
$$= -\frac{GMm}{2R}$$

(b) As the satellite gradually loses energy, its total energy decreases which is more negative, its radius of orbit R decreases. As the satellite radius of orbit R decreases, its kinetic energy increases, resulting in speed increasing and moving nearer into the Earth's atmosphere. The satellite's increasing speed gives rise to increase of resistive forces which results in increasing rate of conversion of its energy to thermal energy, and so the satellite could eventually 'burn up'.

(c)
(i)
(ii)
Advantage: They orbit at the same speed as the Earth's rotation, appearing stationary over one location, providing constant monitoring of a specific area.

Disadvantage: Due to the curvature of the Earth, they have limited coverage of the polar regions.

2

(a) **(i)** work done by an external force per unit mass in bringing a small test mass from infinity to that point

(ii) Potential energy at zero is the max at infinity and gravitational force is an attractive force. By definition of gravitational potential energy, it is negative. Since gravitational potential energy is the product of mass and potential, gravitational potential is negative.

(b) By Conservation of Energy,

$$KE_i + GPE_i = KE_f + GPE_f$$
$$\frac{1}{2}mv^2 + m\phi = 0$$
$$v = \sqrt{-2\phi}$$

(c)
$$\phi = -\frac{GM}{r}$$
$$= \frac{-6.67 \times 10^{-11} \times 7.3 \times 10^{22}}{1.7 \times 10^6}$$
$$= -2.9 \times 10^6 \, J \, kg^{-1}$$

(d)
$$\text{Speed} = \sqrt{2 \times 2.9 \times 10^6)}$$
$$= 2400 \, m \, s^{-1}$$

(e)
$$\frac{1}{2}m < c^2 >= \frac{3}{2}kT$$

$$\frac{1}{2}(3.34\times10^{-27}) < c^2 > = \frac{3}{2}(1.38\times10^{-23})(400)$$
$$c_{rms} = 2200\ m\ s^{-1}$$

(f) there is a distribution of molecular speeds so, many molecules have speeds greater than the escape speed

(g) **(i)** By Conservation of Energy,

$$KE_X + GPE_X = KE_Y + GPE_Y$$
$$\frac{1}{2}mv_X^2 - \frac{GMm}{r_X} = \frac{1}{2}mv_Y^2 - \frac{GMm}{r_Y}$$
$$(6.67\times10^{-11})(1.99\times10^{30})\left[\left(\frac{1}{6.38\times10^{10}}\right)-\left(\frac{1}{8.44\times10^{11}}\right)\right]+\frac{1}{2}(34.1\times10^3)^2 = \frac{1}{2}v_Y^2$$
$$v_Y = 70.8\ km\ s^{-1}$$

(ii) It is independent on m, so path is unchanged.

(iii) Path higher than the original one AND Closest distance of approach is larger. If goes back, the bottom path is lower than the original one.

3

(b) **(i)** To just reach the neutral point from the Earth,
gain in gravitational potential energy GPE = Loss in kinetic energy
$$\Delta U = m\Delta\phi = -\Delta E_K$$
$$10.0\Delta\phi = 6.10\times10^8\ J$$
$$\phi_{neutral\ point} = 6.10\times10^7 + \phi_{earth\ surface}$$
$$\phi_{neutral\ point} = -1.3\times10^6\ J\ kg^{-1}$$

(ii) The rock from the Moon must have enough energy to go past the neutral point, then resultant gravitational force of the Earth-mass system on the mass will accelerate it towards Earth.

gravitational potential difference between the Moon's surface and the neutral point
$$\Delta\phi = -1.30\times10^6 - (-3.90\times10^6)$$
$$= 2.60\times10^6\ J\ kg^{-1}$$

Hence, the minimum kinetic energy needed to send a 1.4 kg rock from the Moon to the neutral point is $m\Delta\phi = (1.4)(2.60\times10^6) = 3.64\times10^6\ J$

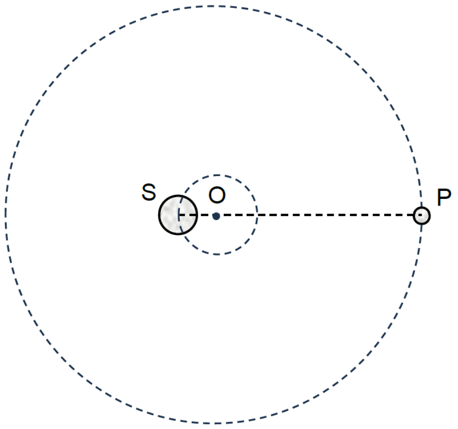
(c) **(i)** The gravitational force exerted on one star by the other star provides the centripetal force for each orbit. This pair of forces is an action-reaction pair by Newton's 3rd law, always equal in magnitude and opposite in direction.

(ii)
$$\omega = \frac{2\pi}{T} = 1.837\times10^{-5}\ rad\ s^{-1}$$
Consider star of mass M
$$F_G = Ma_c$$
$$\frac{GM(2M)}{(3R)^2} = M(2R)\omega^2$$
$$R = \sqrt[3]{\frac{GM}{9\omega^2}}$$

$$= \sqrt[3]{\frac{6.67 \times 10^{-11}(3.14 \times 10^{30})}{9(1.837 \times 10^{-5})^2}}$$
$$= 4.10 \times 10^{9}\ m$$

4

(a) **(i)**



(ii) Star S and planet P attract each other by Newton's law of gravitation. By Newton's third law, the force of attraction on star S by planet P is equal in magnitude and opposite in direction to the force of attraction on P by S. Since the only force acting on each star is this gravitational force of attraction, the gravitational force provides the centripetal force on each star. Hence the magnitude of the centripetal force on each star is the same.

(iii) Since both S and P have the same angular velocity,
$$Mr_S\omega^2 = 0.12Mr_P\omega^2$$
$$\frac{r_S}{r_P} = 0.12$$

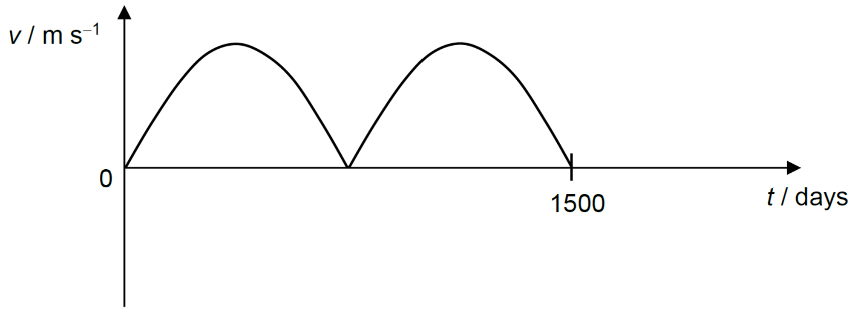
(b) **(i)** Period $T = 1500\ days$
$$\omega = \frac{2\pi}{T} = \frac{2\pi}{1500 \times 24 \times 3600}$$
$$= 4.85 \times 10^{-8}\ rad\ s^{-1}$$

(ii)
$$r_S = \frac{v_S}{\omega}$$
$$= \frac{70}{4.848 \times 10^{-8}} = 1.44 \times 10^{9}\ m$$

(iii)
$$r_P = \frac{r_S}{0.12}$$
$$r_S + r_P = r_S\left(1 + \frac{1}{0.12}\right) = 1.444 \times 10^{9}(9.333) = 1.35 \times 10^{10}\ m$$

(iv) Gravitational force between S and P provides the centripetal force for their circular motion.
$$\frac{GM(0.12M)}{(r_S + r_P)^2} = Mr_S\omega^2$$
$$M = \frac{(r_S + r_P)^2 r_S\omega^2}{0.12G}$$
$$= \frac{(1.348 \times 10^{10})^2 \times 1.444 \times 10^{9} \times (4.848 \times 10^{-8})^2}{0.12 \times 6.67 \times 10^{-11}}$$
$$= 7.71 \times 10^{25}\ kg$$

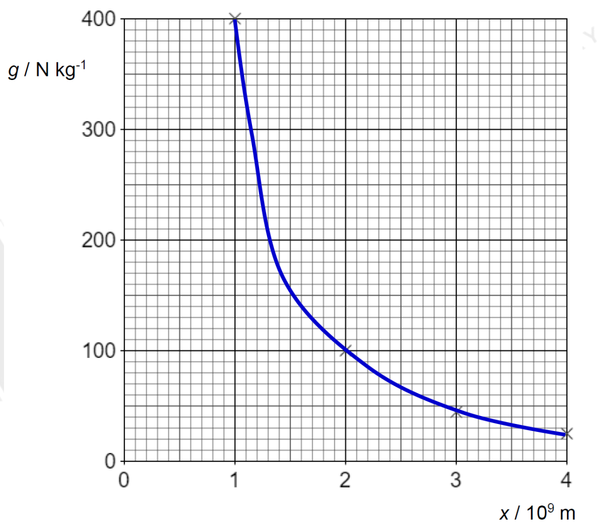
(c)



5

(a)
$$g = \frac{GM}{r^2}$$
$$= \frac{(6.67 \times 10^{-11})(6.0 \times 10^{30})}{(3.0 \times 10^9)^2}$$
$$= 44.5\ N\ kg^{-1}$$

(b)



(c) Change in gravitational potential

6

(a) Similarity: They are both regions of space in which an object can experience a non-contact force.

Difference: Gravitational fields are created by mass while electric fields are created by charge.

(b) Particle A: Gravitational field
Particle B: Electric field
Particle C: Magnetic field

(c) **(i)** The gravitational potential at a point is the work done per unit mass in bringing a small test mass from infinity to that point.

(ii) The magnitude of the gravitational field strength g is numerically equal to the potential gradient. For the same change in gravitational potential between adjacent equipotential lines, the separation between adjacent equipotential lines increases as the distance from the centre the Mars increases.

(iii) By conservation of energy

$$GPE_X + KE_X = GPE_Y + KE_Y$$
$$m(\phi_X - \phi_Y) = \frac{1}{2}mv_Y^2$$
$$v_Y^2 = \sqrt{2(-6.0 + 8.0) \times 10^6}$$
$$= 2000\ m\ s^{-1}$$

7

(a) The gravitational field strength at a point is the gravitational force per unit mass acting on a small test mass placed at that point in the field.

(b) Gravitational force provides the centripetal force.

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$
$$\frac{1}{2}mv^2 = \frac{GMm}{2r}$$
Since $E_P = -\frac{GMm}{2}$
$$E_K = -\frac{E_P}{2}$$

(c) **(i)** From graph, $\phi = -3.2 \times 10^8\ J\ kg^{-1}$ at $r = 4.0 \times 10^8\ m$

$$E_{tot} = E_K + E_P = -\frac{E_P}{2} + E_P = \frac{E_P}{2} = \frac{m\phi}{2}$$
$$= \frac{8.93 \times 10^{22} \times (-3.2 \times 10^8)}{2}$$
$$= -1.43 \times 10^{-31}\ J$$

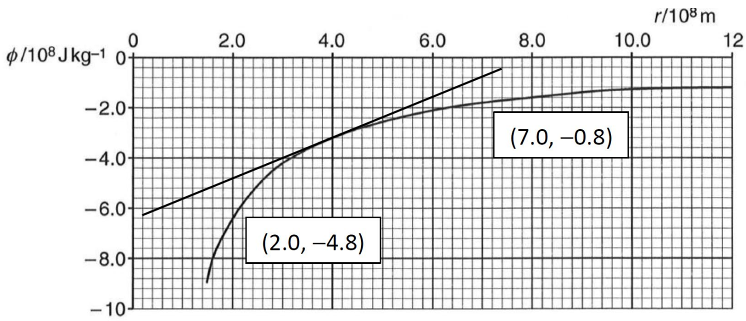
(ii) From **(b)**, $E_K = \frac{GMm}{2r}$

Hence, when r decreases, kinetic energy increases

From **(c)(i)**, $E_{tot} = -\frac{GMm}{2r}$

Hence, when r decreases, total energy becomes more negative and it decreases.

(iii)



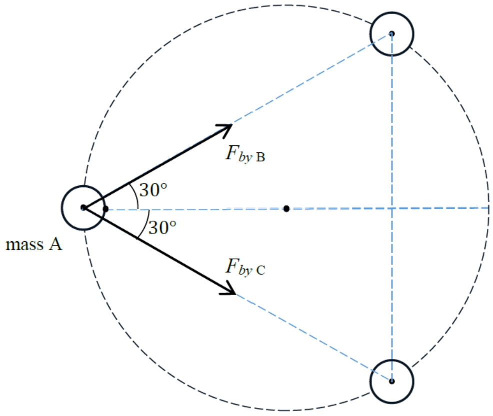
Using (2.0, −4.8) and (7.0, −0.8),

$$g = -\frac{d\phi}{dt}$$
$$= \frac{[(-4.8 - (-0.8)) \times 10^8]}{(2.0 - 7.0) \times 10^8}$$
$$= 0.80\ m\ s^{-2}$$

8

(a) Every point mass attracts every other point mass with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

(b) **(i)**



The vertical components of the two forces due to B and C cancel each other, hence,
Resultant force in the y-direction = 0

Hence, Resultant force F = Resultant force in the x-direction

$$= 2 \times \frac{GM^2}{d^2} \times \cos 30° = 2 \times \frac{(6.67 \times 10^{-11})(6.20 \times 10^{24})^2}{(1.32 \times 10^9)^2} \times \cos 30°$$
$$= 2.55 \times 10^{21}\ N$$

(ii) The resultant gravitational force provides for the centripetal force required for the rotation of planet A.

$$F = mR\omega^2$$
$$\omega = \sqrt{\frac{F}{mR}} = \sqrt{\frac{2.5487 \times 10^{21}}{6.20 \times 10^{24} \times 7.60 \times 10^8}} = 7.3546 \times 10^{-7}\ rad\ s^{-1}$$
$$T = \frac{2\pi}{\omega}$$
$$= 8.54 \times 10^6\ s$$

(iii) The gravitational potential is set to be zero at infinity. Gravitational force is an attractive in nature and the force exerted on a test mass by the external agent will be in opposite direction to the displacement of the mass. Thus negative work is done by the external force to bring a test mass from infinity to the point and hence potential is negative.

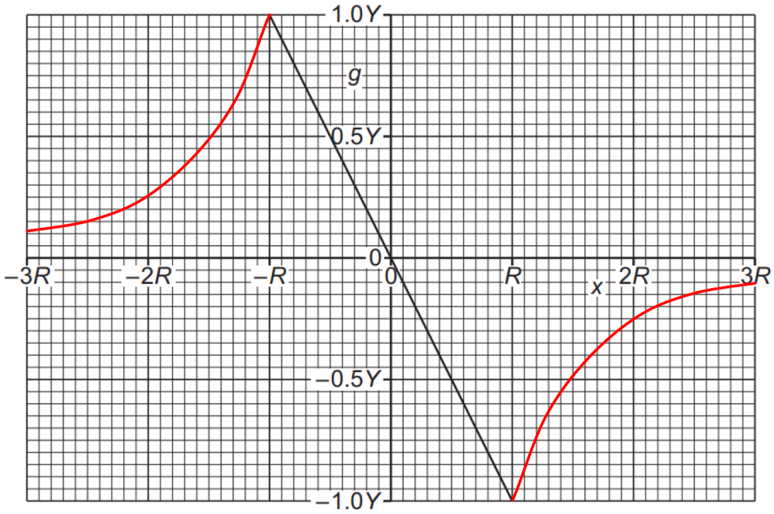
(iv)
$$U = U_{AB} + U_{AC} + U_{BC}$$
$$= -\frac{Gm^2}{d} + \left(-\frac{Gm^2}{d}\right) + \left(-\frac{Gm^2}{d}\right)$$
$$= 3 \times \frac{-6.67 \times 10^{-11}(6.20 \times 10^{24})^2}{1.32 \times 10^9}$$
$$= -5.83 \times 10^{30}\ J$$

9

(a) The change in height is negligible compared with radius of the planet.

(b) **(i)** $Y = GM/R^2$, where G is the gravitational constant

(ii)



(iii) By conservation of energy,

$KE_i + GPE_i = KE_f + GPE_f$

$\frac{1}{2}mv^2 - \frac{GMm}{R} = \frac{1}{2}\left(\frac{1}{2}mv^2\right) - \frac{GMm}{x}$

$\frac{1}{4}mv^2 = GM\left(\frac{1}{R} - \frac{1}{x}\right)$

$\frac{1}{4}(4.7 \times 10^3)^2 = (6.67 \times 10^{-11})(6.4 \times 10^{23})\left(\frac{1}{3.4 \times 10^6} - \frac{1}{x}\right)$

$x = 6.070 \times 10^6\ m$

Distance travelled

$= x - R$

$= 6.070 \times 10^6 - 3.4 \times 10^6$

$= 2.7 \times 10^6\ m$

10

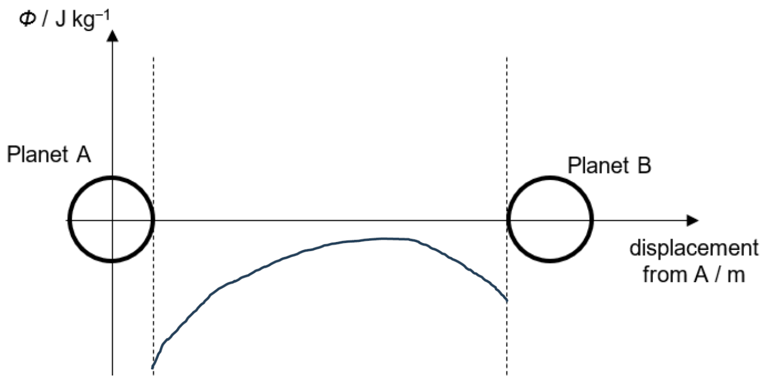
(a) **(i)** The positive direction was defined to be the direction from A to B.

Gravitational field strength $= -\frac{GM_A}{r^2} + \frac{GM_B}{r^2}$

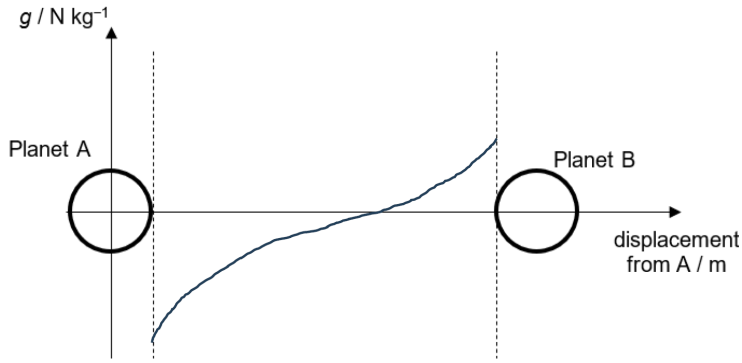
$= \frac{6.67 \times 10^{-11}}{(0.5 \times 3.85 \times 10^8)^2}(-5.07 \times 10^{24} + 3.23 \times 10^{24})$

$= -3.31 \times 10^{-3}\ N\ kg^{-1}$

(ii)



(iii)



(b) **(i)** The same magnitude of force provides for the centripetal force on each star about P.

$$M_C r_C \omega^2 = M_D r_D \omega^2$$
$$\frac{r_C}{r_D} = \frac{M_C}{M_D}$$
$$\frac{r_C}{r_D} = \frac{1}{2}$$

(ii) Gravitational force $= \frac{GM_C\left(\frac{M_C}{2}\right)}{(r_C+2r_C)^2} = \frac{GM_C^2}{18r_C^2}$

(iii) Gravitational force on C by D provides for centripetal force on C

$$\frac{GM_C^2}{18r_C^2} = M_C r_C \omega^2$$
$$M_C = \frac{18r_C^3}{G}\left(\frac{2\pi}{T}\right)^2$$
$$= \frac{18(2.40 \times 10^{12})^3}{6.67 \times 10^{-11}}\left(\frac{2\pi}{3.84 \times 10^9}\right)^2$$
$$= 9.99 \times 10^{30}\ kg$$