1

(a) Rate of change of angular displacement

(b) **(i)** At minimum speed, normal contact force = 0 AND Weight provides centripetal force

$$mg = \frac{mv^2}{r}$$
$$9.81 = \frac{v^2}{0.12}$$
$$v = 1.08\ m\ s^{-1}$$

(ii) Loss in EPE = Gain in GPE + Gain in KE

$$\frac{1}{2}kx^2 = mgh + \frac{1}{2}mv^2$$
$$\frac{1}{2}k(0.16)^2 = (0.076)(9.81)(0.24) + \frac{1}{2}(0.076)(1.08)^2$$
$$k = 17.4\ N\ m^{-1}$$

(c) Loss in KE = Work done against resistive force
$0.23 = (average\ resistive\ force)\ (0.30 + 0.25 + 2\pi(0.12))$
average resistive force = 0.18 N

2

(a) The vertical component of the tension in the spring equal to the weight of the sphere and the horizontal component of the tension provides the centripetal force for circular motion. Since the vertical and horizontal forces combine to give a greater tension in the spring in Fig. 3.2 compared to the tension in Fig 3.1 that equals to the weight of sphere, the spring undergoes a greater extension by Hooke's law.

(b) **(i)**
$$a = r\omega^2 = (0.050)\left(\frac{2\pi}{0.60}\right)^2 = 5.5\ m\ s^{-2}$$

(ii)
$$T\sin\theta = mr\omega^2$$
$$T\cos\theta = mg$$
$$\tan\theta = \frac{r\omega^2}{g} = \frac{(0.050)\left(\frac{2\pi}{0.60}\right)^2}{9.81}$$
$$\theta = 29°$$

(iii)
$$T\cos\theta = mg$$
$$T\cos(29.202) = (0.30)(9.81)$$
$$T = 3.37\ N$$

(iv)
$$\sin\theta = \frac{0.05}{L}$$
$$\sin 29.202 = \frac{0.05}{L}$$
$$L = 0.10248\ m$$
$$k = \frac{\Delta F}{\Delta x} = \frac{3.3715 - (0.30)(9.81)}{0.10248 - 0.085}$$
$$= 24.5\ N\ m^{-1}$$

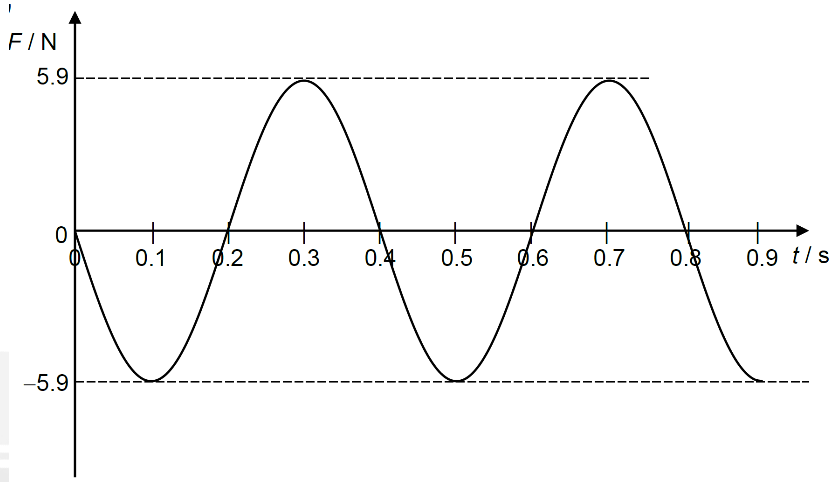
3

(a) The vertical displacement of the yoke is $r\sin(\omega t)$
Hence, the acceleration is $-\omega^2 r\sin(\omega t) = -\omega^2 x$, which is the defining equation for simple harmonic motion.

(b) **(i) 1.**
$$v_0 = \omega y_0$$
$$= \omega r$$
$$= \left(\frac{2\pi}{0.40}\right)(0.080)$$
$$= 1.3\ m\ s^{-1}$$

(i) 2.
$$a_0^2 = \omega^2 y_0$$
$$= \left(\frac{2\pi}{0.40}\right)^2 (0.080)$$
$$= 20\ m\ s^{-2}$$

(ii)



4

(a) Angular velocity is the rate of change of angular displacement. Unit is $rad\ s^{-1}$

(b)
$$mg + T = \frac{mv^2}{r}$$
String just taut implies that $T = 0$
$$mg = \frac{mv^2}{r}$$
$$v = \sqrt{gL}$$

(c) By Conservation of Energy,
Initial KE = Gain in GPE + final KE
$$\frac{1}{2}mV^2 = mg(2L) + \frac{1}{2}mv^2$$
$$\frac{1}{2}mV^2 = mg(2L) + \frac{1}{2}mgL$$
$$V^2 = g(4L) + gL$$
$$V = \sqrt{5gL}$$
$$V = 7.0\ m\ s^{-1}$$

5

(b) **(i)** As the baggage is rotating at constant speed, according to Newton's First Law, a net force is needed to change the direction of motion. This change in direction of motion results in a rate of change of velocity directed towards the centre of rotation. By Newton's Second Law, the net force acts towards the centre of circle

(ii) Considering forces acting perpendicular to the slope

$N = W\cos\theta$
$= (10)(9.81)\cos 37°$
$= 78\ N$

(iii) $f\cos\theta - N\sin\theta = mr\omega^2$

$$mr\left(\frac{2\pi}{T}\right)^2 = f\cos\theta - N\sin\theta$$

$$T = 2\pi\sqrt{\frac{mr}{f\cos\theta - N\sin\theta}}$$

$$T = 2\pi\sqrt{\frac{(10)(10)}{60\cos 37° - 78\sin 37°}}$$

$T = 64\ s$

6

(a) **(i)**

$$F_c = \frac{mv^2}{r}$$
$$= \frac{(1700)(19)^2}{45}$$
$= 13638\ N = 14000N$

(ii) $W - N = 13638$
$(1700)(9.81) - N = 13638$
$N = 3039\ N = 3000\ N$

(b) $W - N = F_c$
$N = W - F_c$

For the car not to lose contact, normal contact force must be more than zero.

$N > 0$
$W - F_c > 0$
$$mg - \frac{mv^2}{r} > 0$$
$v < \sqrt{rg}$
$v < \sqrt{(45)(9.81)}$
$v < 21.011 = 21\ m\ s^{-1}$

7

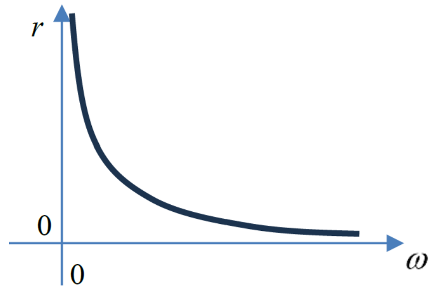
(a) In uniform circular motion, the speed of the metal ball is constant, but its velocity is constantly changing direction. Since acceleration is the rate of change of velocity, the metal ball experiences an acceleration.

(b) **(i)** Horizontal component:
$N\cos\theta = mr\omega^2$
Vertical component:

$N\sin\theta = mg$
$\tan\theta = \dfrac{g}{r\omega^2}$

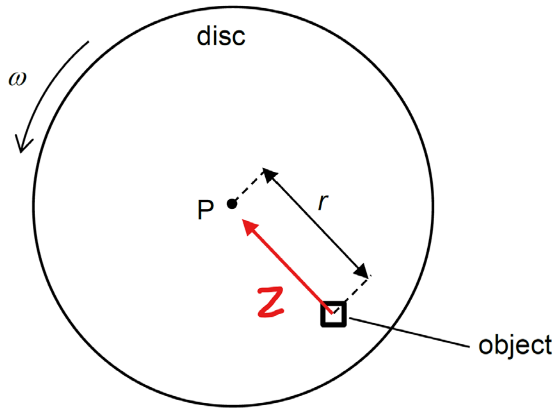
(ii) $\omega = 2\pi f = 2\pi(3) = 6\pi$
$v = r\omega$
$\tan\theta = \dfrac{g}{r\omega^2}$
$= \dfrac{9.81}{0.10(6\pi)^2}$
$\theta = 15°$

(c)



8

(a) **(i)**



(ii) The frictional force provides the centripetal force, and hence must point towards the centre of the disc.

(b) **(i)** $Using\ points\ (6.00, 5.90)\ and\ (9.00, 8.90)\ on\ the\ line$

Gradient $= \frac{8.90-5.90}{9.00-6.00} = 1.00$

(b) **(ii)** Gradient $= \frac{\omega_{max}^2}{\frac{1}{r}} = r\omega_{max}^2 = a_{c_{max}}$

Gradient is numerically equal to the maximum centripetal acceleration.

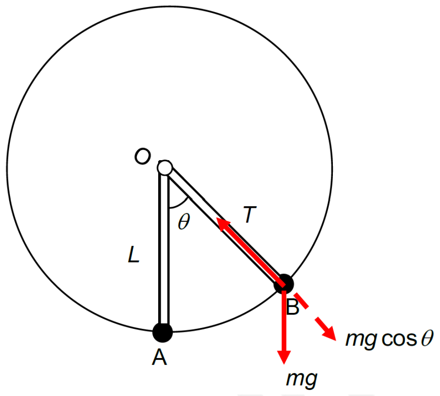
(c) $f = gradient \times m$
$= 1.00 \times 0.80$
$0.80\ N$

(d) As angular speed increases, the required centripetal force increases. When required centripetal force exceeds the maximum frictional force possible, object slides.

9

(a) Since the direction of motion of the body is always changing, its velocity is not constant and thus a resultant force acts on the body.
Since the speed of the body is constant, the resultant force acts perpendicular to its velocity and thus, direction of motion, which is towards the centre of the circle.

(b) **(i)**



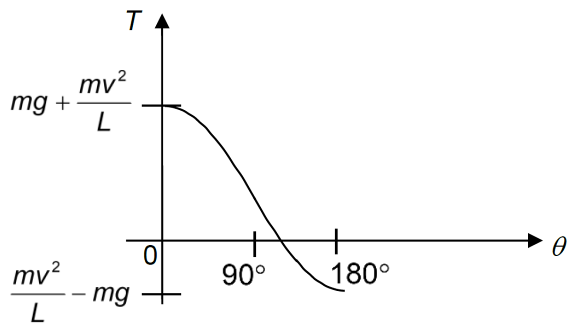
The net force towards centre of the circle provides the centripetal force required to move in a circular path.

$$T - mg\cos\theta = \frac{mv^2}{r}$$
$$T = mg\cos\theta + \frac{mv^2}{L}$$

(ii) The point where the force in the rod changes from tension to compression is when T = 0 N.

$$mg\cos\theta = -\frac{mv^2}{L}$$
$$\cos\theta = -\frac{v^2}{gL} = -\frac{(2.0)^2}{9.81 \times 0.80} = -0.51$$
$$\theta = 120.6° = 120°$$

(iii)



(iv) The K.E. of the mass is constant but its G.P.E. is constantly changing. So as the mass is moving upwards, an external device is needed to supply energy to increase its G.P.E. As the mass moves downwards, its G.P.E. is decreasing and the external device must absorb the lost in G.P.E

10

(a) **(i)** At the top of the circle,

$$T + mg = \frac{mv^2}{L}$$

For the ball to just complete the vertical circle,

$$mg = \frac{mv^2}{L}$$
$$v^2 = gL$$
$$v = \sqrt{gL}$$

(ii) By the principle of conservation of energy, as the ball moves from the top to the bottom of the circle, its gravitational potential energy decreases and its kinetic energy increases. This means the speed at the bottom is greater than the speed at the top. Hence $\frac{v_B}{v_T} > 1$

(iii) $$\frac{v_B}{v_T} = 3$$
$$v_B = 3v_T$$

As the ball moves from the top to the bottom, by Conservation of Energy,

$$\frac{1}{2}mv_B^2 = mg(2L) + \frac{1}{2}mv_T^2$$
$$\frac{9}{2}mv_T^2 - \frac{1}{2}mv_T^2 = 2mgL$$
$$4v_T^2 = 2gL$$
$$v_T = \sqrt{\frac{gL}{2}}$$

Since the v_T is lesser than the $\sqrt{gL}$ needed for the ball required to complete the vertical circular motion at the top, the ratio is not achievable.

(b) Considering the forces along the radial direction,

$$T \sin\theta = mr\omega^2$$
$$r = L\sin\theta$$
$$T\sin\theta = m(L\sin\theta)\omega^2$$
$$T = mL\omega^2$$

Since m and L are constants, $T \propto \omega^2$. When angular velocity doubled the tension is $4T$.