1

(a) Let X be the random variable denoting the mass of a randomly chosen apple in grams.

$$X \sim \mathrm{N}(260, 15^2)$$

$\mathrm{P}(X \geq 250) = 0.748$

(b) $Y \sim \mathrm{N}(210, 10^2)$

$X - Y \sim \mathrm{N}(260 - 210,\ 15^2 + 10^2)$
$X - Y \sim \mathrm{N}(50, 325)$

$\mathrm{P}(|X - Y| > 55)$
$= 1 - \mathrm{P}(-55 \leq X - Y \leq 55)$
$= 0.39076$
$= 0.391$ (to 3 sig fig)

(c) Let $T = 0.95(X_1 + X_2 + \ldots + X_5) + 0.85(Y_1 + Y_2 + \ldots + Y_6)$

$T \sim \mathrm{N}(0.95 \times 5 \times 260 + 0.85 \times 6 \times 210,\ 0.95^2 \times 5 \times 15^2 + 0.85^2 \times 6 \times 10^2)$
$T \sim \mathrm{N}(2306, 1448.8125)$

$\mathrm{P}(T \geq 2300)$
$= 0.56262$
$= 0.563$ (to 3 sig fig)

(d) $W \sim \mathrm{N}(260, 15^2)$ and $V \sim \mathrm{N}(210, 10^2)$

Let $A = 3(0.7W) - 0.8(V_1 + V_2)$

$A \sim \mathrm{N}(0.7 \times 3 \times 260 - 0.8 \times 2 \times 210\ ,\ 0.7^2 \times 3^2 \times (15^2) + 0.8^2 \times 2 \times 10^2)$
$A \sim \mathrm{N}(210, 1120.25)$

$\mathrm{P}(A > 200)$
$= 0.61744$
$= 0.617$(to 3 sig fig)

$\mathrm{P}(3(0.7W) - 0.8(V_1 + V_2) > 200)$ refers to the probability that thrice the cost of a randomly chosen apple exceeds the total cost of two randomly chosen pears by more than \$2.

2

(a) $\mathrm{E}(X+Y)=1+\mu$

$\mathrm{Var}(X+Y)=1+2=3$

$X+Y \sim \mathrm{N}(1+\mu,3)$

$\mathrm{P}(0\le X+Y<3)>0.44$

Using graph on GC,

$-1.1013<\mu<2.1013$

$-1.10<\mu<2.10$

(b) $\mathrm{E}(X_1+X_2+\ldots+X_n-2Y)=n-2(10)=n-20$

$\mathrm{Var}(X_1+X_2+\ldots+X_n-2Y)=n+2^2(2)=n+8$

$X_1+X_2+\ldots+X_n-2Y \sim \mathrm{N}(n-20,n+8)$

$\mathrm{P}(X_1+X_2+\ldots+X_n-2Y\ge 10)<0.03$

From GC,

n	$\mathrm{P}(X_1+X_2+\ldots+X_n-2Y\ge 10)$
19	0.0171
20	0.0294
21	0.0473

So largest n is 20

(c) We need to assume that X and Y are independent.

3

(i) Let X = BMI of 18-year-old boys $X \sim \mathrm{N}\left(22.7, \sigma^2\right)$

$$\mathrm{P}\left(X \leq 16.7\right) = 0.05$$

$$\mathrm{P}\left(Z \leq \frac{16.7 - 22.7}{\sigma}\right) = 0.05$$

$$\frac{-6}{\sigma} = -1.6449$$

$$\sigma = 3.6476 = 3.65 \text{ (3 s.f.)(shown)}$$

(ii) $\mathrm{P}\left(X \geq m\right) \leq 0.1$

Using GC, $m \geq 27.4$

Minimum value of BMI is 27.4

(iii) Let Y = BMI of 15-year-old boys $Y \sim \mathrm{N}\left(21.6, 3.49^2\right)$

$$\frac{X+Y}{2} \sim \mathrm{N}\left(\frac{22.7+21.6}{2}, \frac{3.6476^2 + 3.49^2}{4}\right)$$

$$\frac{X+Y}{2} \sim \mathrm{N}\ (22.15, 6.3713)$$

$$\mathrm{P}\left(\frac{X+Y}{2} > 20.1\right) = 0.792$$

(iv) The BMI of a randomly chosen 15-year-old boy and a randomly chosen 18-year-old boy are **independent** of each other.

4

(i) $\mathrm{P}(\mu-\sigma \le S \le \mu+\sigma) \approx 0.68$

$\mathrm{P}(\mu \le S \le \mu+\sigma) \approx \dfrac{0.68}{2} = 0.34$

(ii)

$\mathrm{P}(S<56)=0.10$

$\mathrm{P}\left(Z<\dfrac{56-\mu}{\sigma}\right)=0.10$

$\dfrac{56-\mu}{\sigma}=-1.2815$

$\mu-1.2815\sigma=56------(1)$

$\mathrm{P}(S>62)=0.30$

$\mathrm{P}\left(Z>\dfrac{62-\mu}{\sigma}\right)=0.30$

$\dfrac{62-\mu}{\sigma}=0.52440$

$\mu+0.52440\sigma=62------(2)$

Using GC,

$\mu=60.2577\approx 60.3$ (3 s.f.)

$\sigma=3.3224\approx 3.32$ (3 s.f.)

(iii) Required Probability $=\mathrm{P}(S_1+S_2+L_1+L_2+L_3\ge 330)$

Let $T=S_1+S_2+L_1+L_2+L_3$

$\mathrm{E}(T)=2(50)+3(75)=325$

$\mathrm{Var}(T)=2(2^2)+3(3.5^2)=44.75$

$T\sim \mathrm{N}(325,44.75)$

$P(T\ge 330)=0.22740=0.227(3\text{ s.f.})$

(iv) Required probability $=\mathrm{P}(3S-1\le L_1+L_2\le 3S+1)$

$=\mathrm{P}(-1\le L_1+L_2-3S\le 1)$

Let $X=L_1+L_2-3S$

$\mathrm{E}(X)=2(75)-3(50)=0$

$\mathrm{Var}(X)=2(3.5^2)+3^2(2^2)=60.5$

$X\sim \mathrm{N}(0,60.5)$

$P(-1\le X\le 1)=0.10229=0.102(3\text{ s.f.})$

(v) Let $\overline{L} = \dfrac{L_1 + \ldots + L_{10}}{10}$.

$\mathrm{E}\left(\overline{L}\right) = 75$

$$\mathrm{Var}\left(\overline{L}\right) = \frac{3.5^2 + 3.5^2 + \ldots + 3.5^2}{10^2} = \frac{10\left(3.5^2\right)}{10^2} = 1.225$$

$\overline{L} \sim \mathrm{N}\left(75, 1.225\right)$

$P\left(\overline{L} < 74\right) = 0.18312 = 0.183\left(3 \text{ s.f.}\right)$

5

(a) Let X denote the mass of a small massage ball in grams.

$X \sim \mathrm{N}(200, \sigma^2)$

P (195 < X < 205) = 0.98273

$$\mathrm{P}\left(\frac{195-200}{\sigma} < Z < \frac{205-200}{\sigma}\right) = 0.98273$$

$$\mathrm{P}\left(-\frac{5}{\sigma} < Z < \frac{5}{\sigma}\right) = 0.98273$$

$$\frac{5}{\sigma} = 2.3809$$

$\sigma = 2.1000 = 2.1$ (1dp)

(b) Let Y denote the mass of a medium massage ball in grams.

$X \sim \mathrm{N}(200, 2.1^2)$ and $Y \sim \mathrm{N}(500, 1.4^2)$

We need $\mathrm{P}(X_1 + \cdots X_6 - 2Y > 210)$

$$\mathrm{E}(X_1 + \cdots X_6 - 2Y) = 6\mathrm{E}(X) - 2\mathrm{E}(Y) = 200$$

$$\mathrm{Var}(X_1 + \cdots X_6 - 2Y) = 6\mathrm{Var}(X) + 4\mathrm{Var}(Y) = 34.3$$

$\therefore X_1 + \cdots X_6 - 2Y \sim N(200, 34.3)$

$\mathrm{P}(X_1 + \cdots X_6 - 2Y > 210) = 0.0439$ (3 s.f.)

(c) Assume that the mass of a massage ball is **independent** of the mass of another massage ball.

6

(a) Let A and S be the mass of a randomly chosen apple from Brand A and Brand S respectively.

$S \sim \mathrm{N}\left(78.8, 3.1^2\right)$ and $A \sim \mathrm{N}\left(82.2, 2.2^2\right)$

Required Probability

$= \mathrm{P}(80 < A < 84)$

$= 0.635$ (3 s.f.)

(b) Let $T = A_1 + A_2 + \ldots + A_5$.

$\mathrm{E}(T) = 5 \times 82.2 = 411$

$\mathrm{Var}(T) = 5 \times 2.2^2 = 24.2$

$T \sim \mathrm{N}(411, 24.2)$

$\mathrm{P}(T > 408) = 0.729$ (3 s.f.)

(c) Let $D = S - 0.9A$.

$\mathrm{E}(D) = 78.8 - 0.9(82.2) = 4.82$

$\mathrm{Var}(D) = 3.1^2 + 0.9^2\left(2.2^2\right) = 13.5304$

$D \sim \mathrm{N}(4.82, 13.5304)$

Required Probability

$= \mathrm{P}(|D| > 1)$

$= 1 - \mathrm{P}(|D| < 1)$

$= 1 - \mathrm{P}(-1 < D < 1)$

$= 0.907$ (3 s.f.)

7

(a) Let X be the mass (kg) of a randomly chosen pumpkin and Y be the mass (kg) of a randomly chosen cabbage.

$X \sim \mathrm{N}\left(3.7, 0.4^2\right) \qquad Y \sim \mathrm{N}\left(0.8, 0.12^2\right)$

$$\mathrm{P}(3.2 < X < m) = 0.6$$

$$\mathrm{P}(X < m) - \mathrm{P}(X < 3.2) = 0.6$$

$$\mathrm{P}(X < m) = 0.70565$$

$$m = 3.9163$$

$$= 3.916 \text{ (3 d.p.) (Shown)}$$

(b) Let W be the number of pumpkins with a mass greater than m kg, out of 20 pumpkins.

$$W \sim \mathrm{B}(20, \mathrm{P}(X > m))$$

$$\mathrm{P}(X > m) = 1 - 0.70565 = 0.29435$$

$$\therefore W \sim \mathrm{B}(20, 0.29435)$$

$$\mathrm{P}(W > 5) = 1 - \mathrm{P}(W \le 5)$$

$$= 0.56178$$

$$= 0.562 \text{ (3 s.f.)}$$

(c) Let $\overline{X} = \dfrac{X_1 + X_2 + \ldots + X_n}{n}$.

$$\overline{X} \sim \mathrm{N}\left(3.7, \frac{0.4^2}{n}\right)$$

$$\mathrm{P}\left(\overline{X} \le 3.8\right) > 0.95$$

From GC,

n	$\mathrm{P}\left(\overline{X} \le 3.8\right)$
43	0.949 (< 0.95)
44	0.951 (> 0.95)
45	0.953 (> 0.95)

Least value of n is 44.

(d) Let S be the total selling price of three randomly chosen pumpkins and four randomly chosen cabbages.

$$S = 5(X_1 + X_2 + X_3) + 3(Y_1 + Y_2 + Y_3 + Y_4)$$

$$\mathrm{E}(S) = 5(3\times 3.7) + 3(4\times 0.8) = 65.1$$

$$\mathrm{Var}(S) = 5^2(3\times 0.4^2) + 3^2(4\times 0.12^2) = 12.5184$$

$$S \sim \mathrm{N}(65.1, 12.5184)$$

$\mathrm{P}(S < 60) = 0.0747$ (3 s.f.)

8

(a)(i) Let B be random variable “diameter of football”.

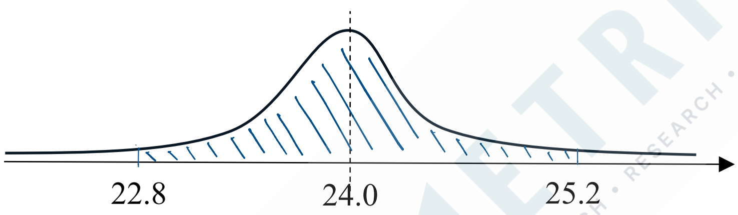
$$B \sim \mathrm{N}(\mu, 0.4^2)$$

$$B - 1.1F \sim \mathrm{N}(\mu - 1.1 \times 22, 0.4^2 + 1.1^2 \times 0.3^2)$$
$$B - 1.1F \sim \mathrm{N}(\mu - 24.2, 0.2689)$$

$$\mathrm{P}(B > 110\% \times F) = 0.35$$
$$\mathrm{P}(B - 1.1F > 0) = 0.35$$
$$\mathrm{P}\left(Z > \frac{0 - (\mu - 24.2)}{\sqrt{0.2689}}\right) = 0.35$$

$$\Rightarrow \frac{0 - (\mu - 24.2)}{\sqrt{0.2689}} = 0.38532$$
$$\therefore \mu = 24.2 - 0.38532 \times \sqrt{0.2689} \approx 24.0$$

(a)(ii)



(iii)

$$B - \overline{F} \sim \mathrm{N}(24 - 22, 0.4^2 + \frac{0.3^2}{10})$$

$$B - \overline{F} \sim \mathrm{N}(2, 0.169)$$

$$\mathrm{P}\left(\left|B - \overline{F}\right| < 1.5\right) = 0.11194 \approx 0.112$$

10(b)(i) Let A be event “player scores on his first attempt” and B be event “player scores on his second attempt”.

$$\mathrm{P}(A) = \frac{5}{11} \qquad \mathrm{P}(B) = \frac{5}{8}$$
$$\mathrm{P}(A \cap B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cup B)$$
$$= \frac{5}{11} + \frac{5}{8} - \mathrm{P}(A \cup B)$$

Since $\mathrm{P}(A \cup B) \le 1 \quad \therefore \mathrm{P}(A \cap B) \ge \frac{7}{88}$

10(b)(ii) $P(A \cap B') = \frac{15}{88}$

$P(A \cup B) = P(A \cap B') + P(B) = \frac{15}{88} + \frac{5}{8} = \frac{35}{44}$

$P(A) + P(B) - P(A \cap B) = \frac{35}{44}$

$P(A \cap B) = \frac{5}{11} + \frac{5}{8} - \frac{35}{44} = \frac{25}{88}$

Since $P(A) \times P(B) = \frac{5}{11} \times \frac{5}{8} = \frac{25}{88} = P(A \cap B)$, the events are independent.

9

(a) Required probability

$= [P(A < 140)]^2 \times P(A > 170) \times \frac{3!}{2!}$

$= [P(A < 140)]^2 \times P(A > 170) \times 3$

$= (0.3341176)^2 \times (0.26015833) \times 3$

$= 0.087128$

$= 0.0871 (3 \text{ s.f})$

(b) Let A be the random variable denoting the mass of an apple from the supermarket.
Let G be the random variable denoting the mass of a guava from the supermarket.

$A \sim N(152, 28^2)$ and $G \sim N(268, 43^2)$

$X = A_1 + A_2 + A_3 + A_4 + A_5$

$$X \sim N(5 \times 152, 5 \times 28^2)$$

$$X \sim N(760, 3920)$$

$Y = G_1 + G_2 + G_3$

$$Y \sim N(3 \times 268, 3 \times 43^2)$$

$$Y \sim N(804, 5547)$$

$$X - Y \sim N(760 - 804, 3920 + 5547)$$

$$X - Y \sim N(-44, 9467)$$

$P(X < Y) = P(X - Y < 0)$

$= 0.6744435$

$= 0.674 \text{ (to 3 s.f.)}$

(c) $F = A_1 + A_2 + A_3 + G_1 + G_2$

$$F \sim N(3 \times 152 + 2 \times 268, 3 \times 28^2 + 2 \times 43^2)$$

$$F \sim N(992, 6050)$$

Given $P(|F - 992| < m) = 0.95$

$P(-m < F - 992 < m) = 0.95$

$P(992 - m < F < 992 + m) = 0.95$

$992 + m = 1144.449$

$m = 152.449$

$m = 153 \text{ (3 s.f.)}$

(d)

$$F \sim N(992, 6050) \text{ (in g)}$$

$$F' \sim N\left(\frac{992}{1000}, \frac{6050}{1000^2}\right) \text{ (in kg)}$$

$$F' \sim \mathrm{N}(0.992, 0.00605)$$

Let C be the cost of a Family Pack ($/kg).

$C = 5F'$

$$C \sim \mathrm{N}(5 \times 0.992, 5^2 \times 0.00605)$$

$$C \sim \mathrm{N}(4.96, 0.15125)$$

$\mathrm{P}(C < 5) = 0.54096$

$= 0.541 \text{(to 3 s.f.)}$

10

(a) Let F and G be the mass of a Fuji and Gala apple respectively

$F \sim \mathrm{N}\left(205, 9^2\right), \; G \sim \mathrm{N}\left(180, 6^2\right)$

$F - G \sim \mathrm{N}\left(205-180,\ 9^2+6^2\right)$

$\quad = \mathrm{N}\left(25,\ 117\right)$

$\mathrm{P}\left(F > G\right) = \mathrm{P}\left(F - G > 0\right)$

$= 0.98960$

$= 0.990$ (3 s.f.)

(b) Required probability

$= \left[\mathrm{P}\left(F > 203\right)\right]^2 \times \mathrm{P}\left(F < 185\right) \times \frac{3!}{2!}$

$= \left(0.58793\right)^2 \times 0.013134 \times 3$

$= 0.013620$

$= 0.0136$ (3 s.f.)

(c) Let A denote the mass of an assorted packet of ten apples.

$A = \left(F_1 + F_2 + \ldots + F_n\right) + \left(G_1 + G_2 + \ldots + G_{10-n}\right)$

$\mathrm{E}\left(A\right) = n\mathrm{E}\left(F\right) + \left(10-n\right)\mathrm{E}\left(G\right)$

$\quad = 205n + \left(10-n\right)180 = 25n + 1800$

$\mathrm{Var}\left(A\right) = n\mathrm{Var}\left(F\right) + \left(10-n\right)\mathrm{Var}\left(G\right)$

$\quad = 9^2 n + \left(10-n\right)\left(6^2\right) = 45n + 360$

$A \sim \mathrm{N}\left(25n+1800,\ 45n+360\right)$

$\therefore A - 9F \sim \mathrm{N}\left(25n + 1800 - 9(205),\ 45n + 360 + 9^2\left(9^2\right)\right)$

$\Rightarrow A - 9F \sim \mathrm{N}\left(25n - 45,\ 45n + 6921\right)$

$\mathrm{P}\left(A - 9F > 28\right) \geq 0.5$

$\mathrm{P}\left(Z > \frac{28 - \left(25n - 45\right)}{\sqrt{45n + 6921}}\right) \geq 0.5$

$28 - \left(25n - 45\right) \leq 0 \quad \Rightarrow n \geq 2.93$

Least n in an assorted packet is 3.

(d) The mass of **every** apple is independent of one another.

11

(a) $W \sim \mathrm{N}(250,\ \sigma^2)$

Standardizing,

$$Z = \frac{W-250}{\sigma} \sim \mathrm{N}(0,\ 1)$$

$$\mathrm{P}(W < 245) = 0.05$$

$$\mathrm{P}\left(Z < \frac{245-250}{\sigma}\right) = 0.05$$

$$\frac{-5}{\sigma} = -1.644853626$$

$$\sigma = 3.039784161$$

$$= 3.0398 \text{ (to 5 s.f.) (shown)}$$

(b) $W_1 + W_2 + \cdots + W_6 \sim \mathrm{N}(1500,\ 55.44172647)$

$F \sim \mathrm{N}(300,\ 2.5^2)$

$5F \sim \mathrm{N}(1500,\ 156.25)$

Let $D = 5F - (W_1 + W_2 + \cdots + W_6)$

$D \sim \mathrm{N}(0,\ 211.6917265)$

$$\mathrm{P}(0 < D \le 20) = 0.4153730399$$

$$= 0.415 \text{ (to 3 s.f.)}$$

(c)

$$\text{Let } M=\frac{\left(W_1+W_2+\cdots+W_n\right)+\left(F_1+F_2+\cdots+F_n\right)}{2n}$$

$$\mathrm{E}(M)=\frac{250n+300n}{2n}=275$$

$$\mathrm{Var}(M)=\frac{(3.039784161)^2 n+(2.5)^2 n}{4n^2}$$

$$=\frac{3.872571936}{n}$$

$$M\sim \mathrm{N}\left(275,\ \frac{3.872571936}{n}\right)$$

$$Z=\frac{M-275}{\sqrt{\frac{3.872571936}{n}}}\sim \mathrm{N}(0,\ 1)$$

$$\mathrm{P}(M\ge 278)<0.015$$

$$\mathrm{P}\left(Z\ge 1.524479216\sqrt{n}\right)<0.015$$

$$\mathrm{P}\left(Z<1.524479216\sqrt{n}\right)>0.985$$

$$1.524479216\sqrt{n}>2.170090375 --- (*)$$

$$n>2.026341439$$

$$n\ge 3$$

$\therefore$ smallest value of $n=3$

12

(a) Let T and L be the random variables denoting the waiting time of a randomly chosen patient to see Doctor Tan and Doctor Lim respectively.

$T \sim \text{N}(30, a^2)$ and $L \sim \text{N}(35, 5^2)$

Given $\text{P}(10 < T < 30) = 0.49379$

$\text{P}(T < 30) - \text{P}(T < 10) = 0.49379$

$\text{P}(T < 10) = \text{P}(T < 30) - 0.49379$

$\text{P}(T < 10) = 0.00621$

Standardizing,

$\text{P}\left(Z < \frac{10-30}{a}\right) = 0.00621$

$-\frac{20}{a} = -2.49998$

$a = 8$

(b) Probability required

$= \text{P}\left(T_1 + T_2 + T_3 + T_4 < 0.9(L_1 + L_2 + L_3)\right)$

$= \text{P}\left(T_1 + T_2 + T_3 + T_4 - 0.9(L_1 + L_2 + L_3) < 0\right)$

$T_1 + T_2 + T_3 + T_4 - 0.9(L_1 + L_2 + L_3) \sim \text{N}(25.5, 316.75)$

$\text{P}\left(T_1 + T_2 + T_3 + T_4 - 0.9(L_1 + L_2 + L_3) < 0\right)$

$= 0.075959 = 0.0760(\text{3s.f.})$

(c) The waiting time of each patient must be **independent** of other patients.

(d) Let k be the minimum duration of waiting time for Doctor Lim's consultation.

$L \sim \text{N}(35, 5^2)$

$\text{P}(L > k) < 0.15$

$k > 40.182$

Hence, $k = 41$ minutes (nearest mins).

(e) Let X be the random variable denoting the number of patients waiting for more than 40 minutes to see Doctor Lim, out of 10.

$X \sim \text{B}(10, \text{P}(L > 35))$

$X \sim \text{B}(10, 0.5)$

$\text{P}(X > 4)$

$= 1 - \text{P}(X \le 4)$

$= 0.62305$

$= 0.623(\text{3sf})$

13

(i) Let M be the random variable denoting the height of a man.

$$M \sim \text{N}(173, 10^2)$$

$$\text{P}(173 - 4 < M < 173 + 4)$$
$$= \text{P}(169 < M < 177)$$
$$= 0.31084 = 0.311 \quad (\text{ to 3 s.f.})$$

(ii) Let W be the random variable denoting the height of a woman.

$$W \sim \text{N}(165, \sigma^2)$$

Given $\text{P}(W < 165) \times \text{P}(W > 160) \times 2! = 0.7$ -----(*)

Since $\text{P}(W < 165) = 0.5$,

$\therefore \text{P}(W > 160) = 0.7$

Standardizing,

$$\text{P}\left(Z > \frac{160 - 165}{\sigma}\right) = 0.7 \text{ where } Z \sim \text{N}(0,1)$$

$$\frac{-5}{\sigma} = -0.5244005$$

$\sigma = 9.5346 = 9.53$ (to 3 s.f.)

(iii) $\text{P}(W > M) = \text{P}(W - M > 0)$

$$\text{E}(W - M) = \text{E}(W) - \text{E}(M) = 165 - 173 = -8$$
$$\text{Var}(W - M) = \text{Var}(W) + \text{Var}(M)$$
$$= 9^2 + 10^2 = 181$$
$$\therefore W - M \sim N(-8, 181)$$

Hence $\text{P}(W - M > 0) = 0.27604 = 0.276$ (to 3 s.f.)

(iv) No, people do not choose their spouses at random.
The heights of a husband and wife may **not be independent**.

(v) $\text{P}(0 < W_1 + W_2 + W_3 - 2M \leq 100)$ ------(#)

Let $A = W_1 + W_2 + W_3 - 2M$

$$\text{E}(A) = 3\text{E}(W) - 2\text{E}(M) = 3(165) - 2(173) = 149$$
$$\text{Var}(A) = 3\text{Var}(W) + 2^2\text{Var}(M)$$
$$= 3(9^2) + 4(10^2) = 643$$

$$\therefore A \sim \text{N}(149, 643)$$

$$\text{P}(0 < W_1 + W_2 + W_3 - 2M \leq 100) = 0.026657 = 0.0267 \text{ (3 s.f)}$$

14

(a) Let C and T be the mass of a randomly chosen chicken and turkey respectively.

$$C \sim N(2.2, 0.5^2)$$

$$T \sim N(10.5, 2.1^2)$$

$P(1 < C < 3.5) = 0.987 \text{ (3 s.f)}$

(b) $P(C > 3.5 | 3.2 < C < 3.7)$

$= \dfrac{P(C > 3.5 \cap 3.2 < C < 3.7)}{P(3.2 < C < 3.7)}$

$= \dfrac{P(3.5 < C < 3.7)}{P(3.2 < C < 3.7)}$

$= 0.155 \text{ (3 s.f)}$

(c) Required probability

$= P(C_1 > 3.5)P(C_2 < 3.5)P(T < 14.5) \times 2!$

$+ P(C_1 < 3.5)P(C_2 < 3.5)P(T > 14.5)$

$+ [P(C < 3.5)]^2 P(T < 14.5)$

$= 0.99971 \text{ (5 s.f)}$

(d) $T - 3C \sim N\left(10.5 - 3(2.2), 2.1^2 + 3^2\left(0.5^2\right)\right)$

$T - 3C \sim N(3.9, 6.66)$

$P\left(|T - 3C| > 0.3\right)$

$= P(T - 3C > 0.3) + P(T - 3C < -0.3)$

$= 1 - P(-0.3 < T - 3C < 0.3)$

$= 0.970$

15

(i) Mass of a completed ornament $Y = 1.05\ (0.9X) = 0.945X$

$Y \sim N(283.5,\ 357.21)$

$P(290 < Y < 350)$
$= 0.365238$
$= 0.365$ (3 s.f.)

(ii) Let X be the number of ornaments, out of 9 that has mass between 290 g and 350 g.

$X \sim B(9, 0.365)$

Required probability
= Out of the first 9 ornaments, exactly 2 has mass between 290 g and 350 g × 10^{th} ornament has mass between 290 g and 350 g

$= P(X = 2) \times 0.365$

$= 0.072878121$

$= 0.0729$ (3 s.f.)

(iii) Let B be the mass of a box.

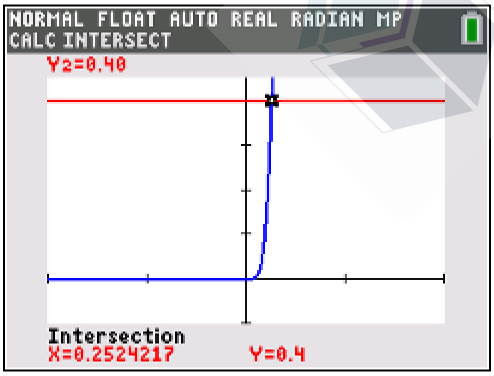
$B = \alpha Y \sim N(283.5\alpha,\ 357.21\alpha^2)$

Let S be the total mass of an ornament and its box.

$S = Y + B \sim N(283.5\alpha + 283.5,\ 357.21\alpha^2 + 357.21)$

$S \sim N(283.5(\alpha + 1),\ 357.21(\alpha^2 + 1))$

Given $P(S > 360) = 0.4$



By GC, $\alpha = 0.252$

If $W \sim N(130,\ 80^2)$, $P(W < 0) \approx 0.0521$

That is, approximately 5.21% of the blocks are of negative masses. Thus, this distribution is not appropriate.

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(a) $X \sim \mathrm{N}(\mu, \sigma^2)$

$\mathrm{P}(X > \mu + 0.05\sigma)$

$= \mathrm{P}\left(\frac{X-\mu}{\sigma} > \frac{\mu + 0.05\sigma - \mu}{\sigma}\right)$

$= \mathrm{P}(Z > 0.05)$

$= 0.480061126$

$= 0.480$ (3 s.f.)

(b) $X \sim \mathrm{N}(\mu, \sigma^2)$

$\overline{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$

$\mathrm{P}(|\overline{X} - \mu| \geq 0.05\sigma) < 0.4$

$P\left(\frac{|\overline{X}-\mu|}{\frac{\sigma}{\sqrt{n}}} \geq \frac{0.05\sigma}{\frac{\sigma}{\sqrt{n}}}\right) < 0.4$

$P\left(|Z| \geq 0.05\sqrt{n}\right) < 0.4$

$1 - P\left(-0.05\sqrt{n} \leq Z \leq 0.05\sqrt{n}\right) < 0.4$

<u>Using GC</u>,

n	$1 - P\left(-0.05\sqrt{n} \leq Z \leq 0.05\sqrt{n}\right)$	
283	0.4003	> 0.4
284	0.3994	< 0.4
285	0.3986	< 0.4

Least $n = 284$

(c) Let Y be the number of observations, out of m, exceed $\mu + 0.05\sigma$

$Y \sim \mathrm{B}(m,\ 0.48006)$

$\mathrm{P}(Y \geq 4) \leq 0.95$

$1 - \mathrm{P}(Y \leq 3) \leq 0.95$

Using GC,

M	$1 - \mathrm{P}(Y \leq 3)$	
12	0.9057	< 0.95
13	0.9382	< 0.95
14	0.9601	>0.95

Greatest m = 13

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(a) $X \sim N(m,\ 3.7)$.

$E(3X_1 - X_2) = 3E(X) - E(X) = 2m$

$Var(3X_1 - X_2) = 9Var(X) + Var(X) = 37$

$\therefore 3X_1 - X_2 \sim N(2m,\ 37)$

$P(-6 < 3X_1 - X_2 < 0)$ is greatest when $E(3X_1 - X_2) = \dfrac{-6+0}{2} = -3$

$\therefore 3X_1 - X_2 \sim N(-3,\ 37)$

Greatest $P(-6 < 3X_1 - X_2 < 0) = 0.378$ (3 s.f.)

(b) $\overline{X} \sim N(m,\ \frac{3.7}{n})$

$P(\overline{X} > 1) \leq 0.01$

Standardizing,

$P(Z > \dfrac{1-m}{\sqrt{\frac{3.7}{n}}}) \leq 0.01$

$\dfrac{1-m}{\sqrt{\frac{3.7}{n}}} \geq 2.3263$

$1 - m \geq 2.33\sqrt{\dfrac{3.7}{n}}$

$(1-m)\sqrt{n} \geq 4.47$