1

(a)(i) Number of ways = ${}^{26}C_6 \times {}^{3}C_1 \times 6! = 497296800$

(a)(ii) Case 1: 3 letters with two different colours each

Number of ways = ${}^{26}C_3 \times \left({}^{3}C_2\right)^3 \times 6! = 50544000$

Case 2: 1 letter with 3 different colours and 1 letter with 2 colours and 1 letter with 1 colour

Number of ways = ${}^{26}C_3 \times {}^{3}C_2 \times {}^{3}C_1 \times 3! \times 6! = 101088000$

Total number of ways = $50544000 + 101088000 = 151632000$

(b)(i)

$$\frac{2+c}{41+c} \times \frac{1+c}{40+c} = \frac{1}{66} \quad ---- \quad (1)$$

$$66(2+c)(1+c) = (41+c)(40+c)$$

$$65c^2 + 117c - 1508 = 0$$

$$c = 4 \quad \text{or} \quad -\frac{29}{5} \text{ (Rejected)}$$

(b)(ii) Case 1: red flower and non-red bead

$$\frac{6}{45} \times \frac{28}{44} \times 2 = \frac{28}{165}$$

Case 2: red non-flower and non-red flower

$$\frac{11}{45} \times \frac{5}{44} \times 2 = \frac{1}{18}$$

Required probability = $\dfrac{28}{165} + \dfrac{1}{18} = \dfrac{223}{990}$

2

(a)(i) No. of ways $= {}^{12}C_3 \times 3! = 1320$

$$\text{No. of ways} = \underbrace{{}^{12}C_2 \times {}^{6}C_1 \times 3!}_{\substack{\text{Select 2 girls from 12 \& 1 boy from 6}\\ \text{followed by arrangement}}} + \underbrace{{}^{12}C_1 \times {}^{6}C_2 \times 3!}_{\substack{\text{Select 1 girl from 12 \& 2 boys from 6}\\ \text{followed by arrangement}}}$$

$$= 3456$$

(b) $$\text{Required probability} = \frac{(15-1)! \times {}^{15}C_3 \times 3!}{(18-1)!}$$

$$= \frac{91}{136}$$

(c) $$\text{Required probability} = \frac{(6-1)! \times 12!}{(18-1)!}$$

$$= \frac{1}{6188}$$

3

(i) Required number of ways $= {}^{20}C_4 \times 4!$

$= 116280$

(ii)

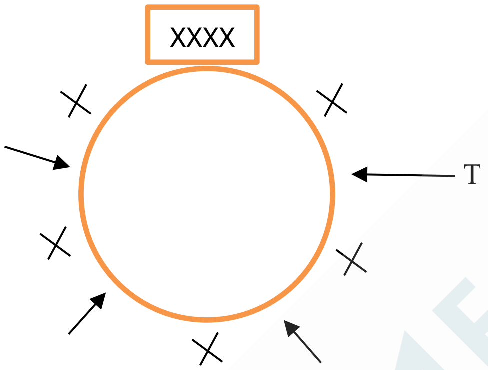
Number of ways for all males $= {}^{12}C_4 \times 4! = 11880$

Number of ways for all females $= {}^{8}C_4 \times 4! = 1680$

Required number of ways $= 116280 - 11880 - 1680$

$= 102720$

(iii)



$$\text{Required Probability} = \frac{(6-1)! \times {}^{4}C_1 \times 4!}{(10-1)!}$$

$= 0.031746$

$= 0.0317$ (3 s.f.)

4

(a) $^{3}C_{2} \times ^{4}C_{2} \times ^{5}C_{2} = 180$

(b) $\frac{(4-1)! \times ^{4}C_{2} \times 2!}{(6-1)!} = \frac{3}{5}$

OR

$1 \quad \frac{(5-1)! \times 2!}{(6-1)!} = \frac{3}{5}$

(c) $^{3}C_{3} \times ^{4}C_{2} \times ^{5}C_{2} + ^{3}C_{2} \times ^{4}C_{3} \times ^{5}C_{2} + ^{3}C_{2} \times ^{4}C_{2} \times ^{5}C_{3}$

$= 60 + 180 + 120$

$= 360$

(d) $\frac{3! \times 2! \times 2! \times 3!}{7!}$

$= \frac{1}{35}$

5

(a) No. of ways choosing 3 male and 3 female artistes

$= {}^{7}C_{3} \times {}^{5}C_{3} = 350$

No of pairing up the chosen = 3!

No. of ways of arranging the order of the 3 pairs = 3!

Total number of ways = 350 ×3!×3! = 12600

(b)

P(even no.) = $\frac{1}{45}(2+4+6+8) = \frac{20}{45}$

P(odd no.) = $\frac{1}{45}(1+3+5+7+9) = \frac{25}{45}$

P(Pair A loses a game) = P(even & black) + P(odd & black)

$$= \frac{20}{45}\left(\frac{3}{8}\right) + \frac{25}{45}\left(\frac{8}{12}\right) = \frac{29}{54}$$

(c) P(Pair B wins)

$$= \left(\frac{29}{54}\right)\left(\frac{25}{54}\right) + \left(\frac{29}{54}\right)^3\left(\frac{25}{54}\right) + \left(\frac{29}{54}\right)^5\left(\frac{25}{54}\right) + \cdots$$

$$= \left(\frac{29}{54}\right)\left(\frac{25}{54}\right)\left[1 + \left(\frac{29}{54}\right)^2 + \left(\frac{29}{54}\right)^4 + \cdots\right]$$

$$= \left(\frac{29}{54}\right)\left(\frac{25}{54}\right)\left[\frac{1}{1-\left(\frac{29}{54}\right)^2}\right]$$

$$= \frac{29}{83}$$

6

(a) Probability

$= \frac{1}{9}.\frac{2}{6} + \frac{3}{9}.\frac{3}{6} + \frac{5}{9}.\frac{1}{6}$

$= \frac{16}{54} = \frac{8}{27}$

(b) Probability

$= \frac{\frac{1}{9}.\frac{2}{6}}{\frac{8}{27}} = \frac{1}{8}$

(c) To win \$6 in total for 3 games, each game he must win \$2.

$= \frac{1}{9}.\frac{2}{6} \times \frac{3}{9}.\frac{3}{6} \times \frac{5}{9}.\frac{1}{6} \times 3!$

$= \frac{540}{157464}$

$= \frac{5}{1458}$ or 0.00343 (3 s.f.)

(d) The participant should end the game by taking the first option because if he proceeds to throw the die, there is only a one-sixth chance that he will take home a higher amount.

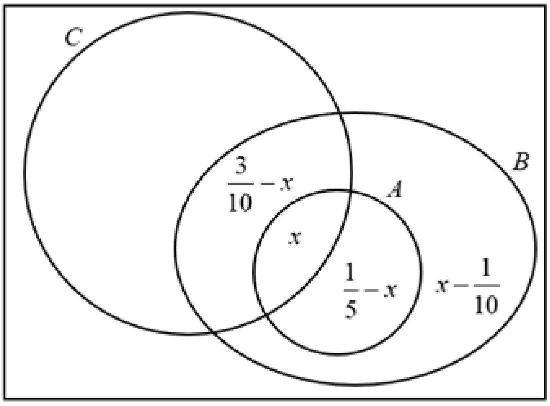
OR

$\frac{2}{6}(0.02) + \frac{3}{6}(0.5) + \frac{1}{6}(2) = 0.59$

For the throw of die, the expected factor is 0.59 which is less than 1. This means that the participant is unlikely to take home a higher amount if he were to proceed to throw the die.

7

(a) Let $\mathrm{P}(A \cap C) = x$.



$\mathrm{P}(A \cap C') = \frac{1}{5} - x$

$\mathrm{P}(A' \cap B \cap C) = \frac{3}{10} - x$

$\mathrm{P}(A' \cap B \cap C') = 2\left(\frac{1}{5}\right) - \frac{1}{5} - \left(\frac{3}{10} - x\right) = x - \frac{1}{10}$

$\frac{1}{5} - x \geq 0$ and $\frac{3}{10} - x \geq 0$ and $x - \frac{1}{10} \geq 0$

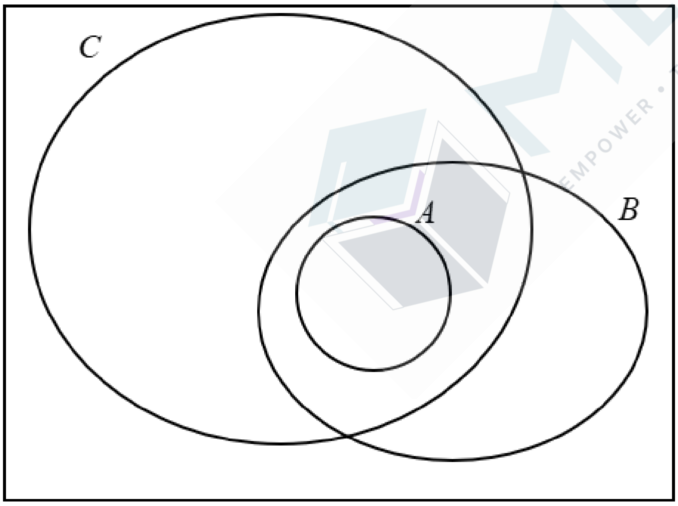
$x \leq \frac{1}{5}$ and $x \leq \frac{3}{10}$ and $x \geq \frac{1}{10}$

Hence, $\frac{1}{10} \leq x \leq \frac{1}{5}$.

Greatest value of $\mathrm{P}(A \cap C) = \frac{1}{5}$

Least value of $\mathrm{P}(A \cap C) = \frac{1}{10}$

(b)



(c) $\mathrm{P}(A) = \frac{1}{5}$

$P(B' \cap C) = 1 - \frac{1}{12} - 2\left(\frac{1}{5}\right) = \frac{31}{60}$

$\mathrm{P}(C) = \frac{31}{60} + \frac{3}{10} = \frac{49}{60}$

For A and C to be independent,

$\mathrm{P}(A \cap C) = \mathrm{P}(A) \times \mathrm{P}(C) = \frac{1}{5} \times \frac{49}{60} = \frac{49}{300}$

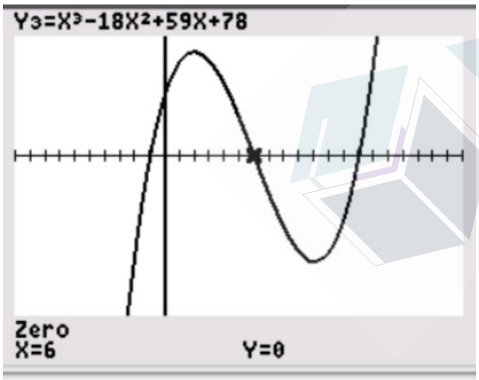
8

(a)(i) P(A wins)

$$=\left(\frac{a-1}{8}\right)\left(\frac{a-2}{8}\right)+\left(\frac{a-1}{8}\right)\left(\frac{10-a}{8}\right)\left(\frac{a-3}{8}\right)+$$
$$\left(\frac{9-a}{8}\right)\left(\frac{a-2}{8}\right)\left(\frac{a-3}{8}\right)$$
$$=\frac{a^2-3a+2}{8^2}+\frac{-a^3+14a^2-43a+30}{8^3}+$$
$$\frac{-a^3+14a^2-51a+54}{8^3}$$
$$=\frac{8a^2-24a+16}{8^3}+\frac{-2a^3+28a^2-94a+84}{8^3}$$
$$=\frac{-2a^3+36a^2-118a+100}{512}$$
$$=\frac{-a^3+18a^2-59a+50}{256}$$

(a)(ii) For the match to be fair,

$$\frac{-a^3+18a^2-59a+50}{256}=\frac{1}{2}$$
$$a^3-18a^2+59a+78=0$$



From the GC, $a = -1,\ 6$ or 13

$a = 6$

Since $a > 0$, we reject $a = -1$.

When $a = 13$, for $k = 1,\ 2,\ 3$, we have $10 \le a-k \le 12$

$$\frac{10}{8} \le \frac{a-k}{8} \le \frac{12}{8}$$

Since probability > 1, we reject $a = 13$.

When a = 6, for $k = 1,\ 2,\ 3$, we have $3 \le a-k \le 5$.

$$\frac{3}{8} \le \frac{a-k}{8} \le \frac{5}{8}$$

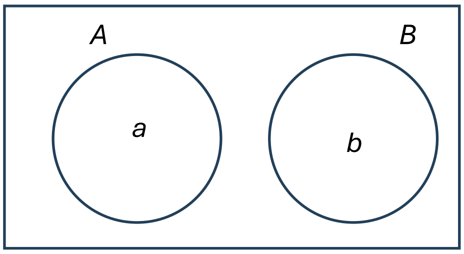
(b) Using part (a) with substitution of a = 7,

P(A wins 2[nd] set | A wins the match)

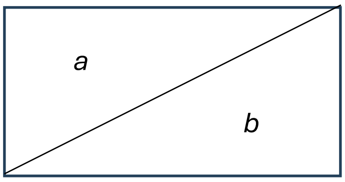
$$=\frac{\frac{6}{8}\times\frac{5}{8}+\frac{2}{8}\times\frac{5}{8}\times\frac{4}{8}}{\frac{176}{256}}$$
$$=\frac{35}{44}=0.795 \text{ (3 s.f.)}$$

9

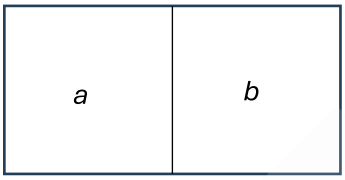
(i) $P(A' \cap B') = 1 - P(A) - P(B) = 1 - a - b$



(ii)



or

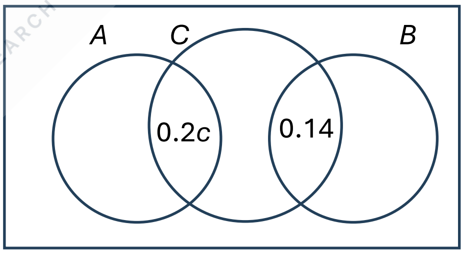


If A' and B' are mutually exclusive events, then

$P(A' \cap B') = 1 - a - b = 0$

$a + b = 1$

(iii) Since A and C are independent events, $P(A \cap C) = ac$



$P(A \cup B \cup C)$

$= P(A) + P(B) + P(C) - P(A \cap C) - P(B \cap C)$

$= a + b + c - ac - 0.14$

$= 0.2 + 0.3 + c - 0.2c - 0.14$

$= 0.36 + 0.8c$

(iv) $P(A' \cap B' \cap C)$

$= P(A \cup B \cup C) - P(A) - P(B)$

$= 0.36 + 0.8c - 0.2 - 0.3$

$= 0.8c - 0.14$

(v) $P(A \cup B \cup C) = 0.36 + 0.8c \le 1$ and $P(A' \cap B' \cap C) = 0.8c - 0.14 \ge 0$

$0.8c \le 0.64$ | $0.8c \ge 0.14$

$c \le 0.8$ | $c \ge 0.175$

$0.175 \le c \le 0.8$

10

(i) Since A and B are mutually exclusive,
$\mathrm{P}(A \cap B) = 0$

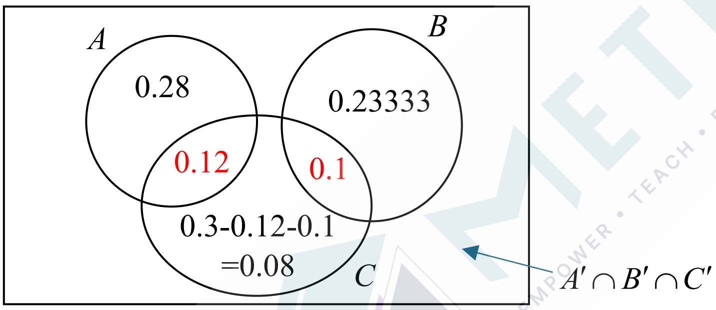
(ii) Since A and C are independent,
$\mathrm{P}(A \mid C) = \mathrm{P}(A) = 0.4$

(iii) Since A and C are independent,

$$\begin{aligned}\mathrm{P}(A \cap C) &= \mathrm{P}(A) \times \mathrm{P}(C)\\ &= 0.4 \times 0.3\\ &= 0.12\end{aligned}$$

Since B and C are independent,

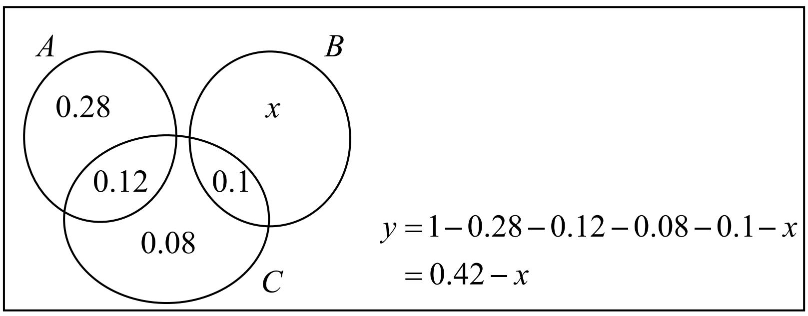
$$\begin{aligned}\mathrm{P}(B) \times \mathrm{P}(C) &= \mathrm{P}(B \cap C)\\ \mathrm{P}(B) \times 0.3 &= 0.1\\ \mathrm{P}(B) &= \frac{1}{3}\end{aligned}$$



From Venn diagram,

$$\begin{aligned}\mathrm{P}(A' \cap B' \cap C') &= 1 - 0.4 - \frac{1}{3} - 0.08\\ &= 0.18667\\ &= 0.187 \text{ (3 s.f.)}\end{aligned}$$

(iv)



Let $\mathrm{P}(\mathrm{B}) = 0.1 + x$

For least $\mathrm{P}(B)$, let $x = 0$. So least $\mathrm{P}(B) = 0.1$

Let $\mathrm{P}(A' \cap B' \cap C') = y$.

$$\begin{aligned}y &= 1 - 0.28 - 0.12 - 0.08 - 0.1 - x\\ &= 0.42 - x\end{aligned}$$

For greatest $\mathrm{P}(B)$, let $y = 0$.

Then $0.42 - x = 0 \Rightarrow x = 0.42$

So greatest $\mathrm{P}(B) = 0.1 + 0.42 = 0.52$

11

(a)

$$P(A \mid B) = P(A) = 0.1$$

$$P(A \mid C) = \frac{P(A \cap C)}{P(C)} = \frac{0}{c} = 0$$

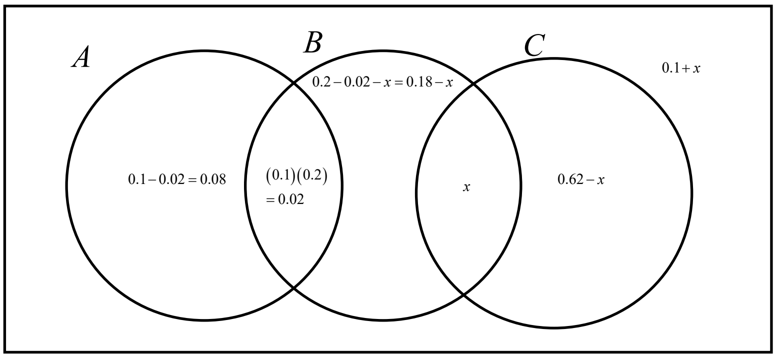
(b)

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= P(A) + P(B) - P(A)P(B) \\ &= 0.1 + 0.2 - 0.1(0.2) \\ &= 0.28 \end{aligned}$$

$$\begin{aligned} P(A' \cap C') &= 1 - P(A \cup C) \\ &= 1 - \left[0.1 + P(C) - P(A \cap C)\right] \\ &= 1 - (0.1 + c - 0) \\ &= 0.9 - c \end{aligned}$$

$$\begin{aligned} P(A \cup B) &= P(A' \cap C') \\ 0.28 &= 0.9 - c \\ c &= 0.62 \end{aligned}$$

(c)



$c = \mathrm{P}(C) = 0.8 - 0.9b = 0.8 - 0.9(0.2) = 0.62$

Let $\mathrm{P}(B \cap C) = x$

$\mathrm{P}(A \cup B \cup C)$

$= \mathrm{P}(A) + \mathrm{P}(B) + \mathrm{P}(C)$

$- \mathrm{P}(A \cap B) - \mathrm{P}(A \cap C) - \mathrm{P}(B \cap C) + \mathrm{P}(A \cap B \cap C)$

$= 0.1 + 0.2 + 0.62 - (0.1)(0.2) - 0 - x + 0$

$= 0.9 - x$

$\mathrm{P}(A' \cap B' \cap C')$

$= 1 - \mathrm{P}(A \cup B \cup C)$

$= 1 - (0.9 - x)$

$= 0.1 + x$

$\mathrm{P}(B) = \mathrm{P}(A \cap B) + \mathrm{P}(B \cap C) + \mathrm{P}(B \cap A' \cap C')$

$\mathrm{P}(B \cap A' \cap C')$

$= \mathrm{P}(B) - \mathrm{P}(A \cap B) - \mathrm{P}(B \cap C)$

$= 0.2 - 0.02 - x$

$= 0.18 - x$

$\mathrm{P}(B \cap C) \geq 0 \Rightarrow x \geq 0$

$\mathrm{P}(B \cap A' \cap C') \geq 0 \Rightarrow 0.18 - x \geq 0 \Rightarrow x \leq 0.18$

$\Rightarrow 0 \leq x \leq 0.18$

$\Rightarrow 0.1 \leq x + 0.1 \leq 0.28$

$\therefore 0.1 \leq \mathrm{P}(A' \cap B' \cap C') \leq 0.28$

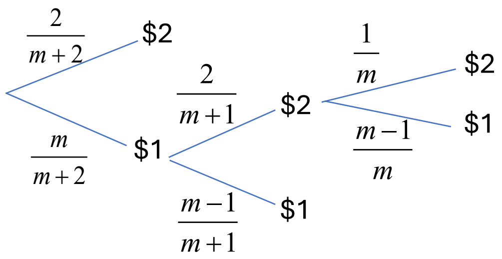
12

(a)

$$\text{P}(\text{1st person has \$2 note}) = \frac{1}{m+1}$$

$$\therefore \text{ Required prob } = 1 - \frac{1}{m+1} = \frac{m}{m+1}$$

(b)(i)



$$\text{Required prob } = \frac{m}{m+2}\times\frac{m-1}{m+1} + \frac{m}{m+2}\times\frac{2}{m+1}\times\frac{m-1}{m}$$

$$= \frac{m^2+m-2}{(m+1)(m+2)}$$

$$= \frac{(m-1)(m+2)}{(m+1)(m+2)}$$

$$= \frac{m-1}{m+1} \quad \text{(shown)}$$

(b)(ii)

P(Vendor unable to sell tickets to the first three people in the queue) $= 1 - \dfrac{m-1}{m+1}$

$$= \frac{2}{m+1}$$

$$\text{P}(\$1, \$2, \$2) = \frac{m}{m+2}\times\frac{2}{m+1}\times\frac{1}{m} = \frac{2}{(m+2)(m+1)}$$

$$\therefore \text{ Required prob } = \frac{\frac{2}{(m+2)(m+1)}}{\frac{2}{m+1}} = \frac{1}{m+2}$$

13

(a) Given that A and B are independent events,

$P(A \cap B) = P(A)P(B) = ab$

Since $P(A' \cap B') = 1 - P(A \cup B)$

$$= 1 - [P(A) + P(B) - P(A \cap B)]$$

$$= 1 - [a + b - ab]$$

$$= (1-a)(1-b)$$

$$= P(A')P(B')$$

$\therefore$ A' and B' are independent events. (shown)

(b) $P(A \cup B) = 0.7$

$P(A) + P(B) - P(A \cap B) = 0.7$

$a + b - ab = 0.7$

$a + 0.5 - 0.5a = 0.7$

$0.5a = 0.2$

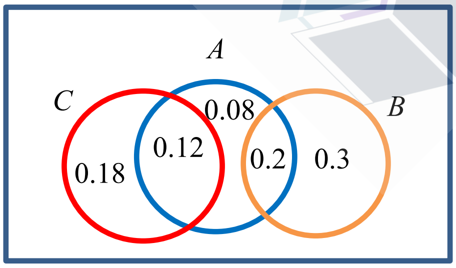
$a = 0.4$ (shown)

(c) Given $P(A) = 0.4, P(B) = 0.5, P(C) = 0.3$

Since A and B are independent events, $P(A \cap B) = 0.4 \times 0.5 = 0.2$

Since A and C are independent events, $P(A \cap C) = 0.4 \times 0.3 = 0.12$

Events B and C are mutually exclusive:



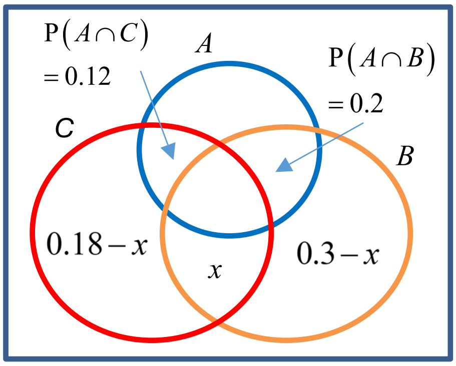
$P(A' \cap B' \cap C') = 1 - P(A) - 0.18 - 0.3 = 1 - 0.4 - 0.18 - 0.3 = 0.12$

(d) Given $P(A) = 0.4, P(B) = 0.5, P(C) = 0.3$

Since A and B are independent events, $P(A \cap B) = 0.4 \times 0.5 = 0.2$

Since A and C are independent events, $P(A \cap C) = 0.4 \times 0.3 = 0.12$

Given events B and C are not mutually exclusive,



$P(A' \cap B' \cap C') = 1 - P(A) - 0.18 - (0.3 - x) = 0.12 + x$

Since probabilities are all non-negative, $0 \le x \le 0.18$.

When x is greatest, i.e. when $x = 0.18$, $P(A' \cap B' \cap C')$ is greatest.

$\therefore$ Greatest possible value of $P(A' \cap B' \cap C')$ is $0.12 + 0.18 = 0.3$.

14

(i)

$$\begin{aligned}
&\mathrm{P}(\text{Get all Bots} \cap \text{Get all number sets}) \\
&= \mathrm{P}(\text{box contain } \alpha \text{ and } \mathbb{R}) \times \mathrm{P}(\text{box contain } \beta \text{ and } \mathbb{Z}) \\
&\qquad \times \mathrm{P}(\text{box contain } \gamma \text{ and } \mathbb{Q}) \times \mathrm{P}(\text{box contain } \omega \text{ and } \mathbb{N}) \\
&\qquad + \mathrm{P}(\text{box contain } \alpha \text{ and } \mathbb{Z}) \times \mathrm{P}(\text{box contain } \beta \text{ and } \mathbb{Q}) \\
&\qquad \times \mathrm{P}(\text{box contain } \gamma \text{ and } \mathbb{R}) \times \mathrm{P}(\text{box contain } \omega \text{ and } \mathbb{N}) \\
&= \left(\frac{1}{8} \times \frac{1}{8} \times \frac{1}{8} \times \frac{1}{10}\right)(4!) + \left(\frac{1}{8} \times \frac{1}{8} \times \frac{1}{8} \times \frac{1}{10}\right)(4!) \\
&= \frac{24}{2560} \\
&= \frac{3}{320} \text{ (shown)}
\end{aligned}$$

(ii)

$$\begin{aligned}
&\mathrm{P}(\text{Get all bots} \mid \text{get all number sets}) \\
&= \frac{\mathrm{P}(\text{Get all Bots} \cap \text{Get all number sets})}{\mathrm{P}(\text{Get all number sets})} \\
&= \frac{\frac{3}{320}}{\frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times 4!} \\
&= \frac{1}{10}
\end{aligned}$$

(iii)

$$\begin{aligned}
\mathrm{P}(\text{get all bots}) &= 0.3 \times 0.3 \times 0.3 \times 0.1 \times 4! \\
&= 0.0648 \\
&\neq 0.1 \\
&= \mathrm{P}(\text{get all bots} \mid \text{get all number sets})
\end{aligned}$$

The event that he gets all the bots and the event that he get all the number sets are not independent.

OR

$$\mathrm{P}(\text{Get all bots} \cap \text{Get all number sets}) = \frac{3}{320}$$

$$\begin{aligned}
&\mathrm{P}(\text{Get all bots}) \times \mathrm{P}(\text{Get all number sets}) \\
&= \left(0.3^3 \times 0.1 \times (4!)\right)\left(\frac{1}{4^4} \times (4!)\right) = 0.006048
\end{aligned}$$

$$\begin{aligned}
&\mathrm{P}(\text{Get all bots} \cap \text{Get all number sets}) \\
&\neq \mathrm{P}(\text{Get all bots}) \times \mathrm{P}(\text{Get all number sets})
\end{aligned}$$

The event that he gets all the bots and the event that he get all the number sets are not independent

15

(i) P(toy chosen is either a Triangle or a Star given not Yellow)

$$=\frac{\text{P(Triangle or Star and not Yellow)}}{\text{P(not Yellow)}}$$

$$=\frac{\frac{(4+2+3)+(5+3+1)}{40}}{\frac{(40-1-3-4-2)}{40}}=\frac{18}{30}=\frac{3}{5}$$

(ii)(a) P(both toys chosen are purple and different shapes)

= P(Purple Square, Purple Triangle)

\+ P(Purple Star, Purple Triangle)

+P(Purple Square, Purple Star)

$$=2\times\frac{2}{40}\times\frac{3}{39}+2\times\frac{1}{40}\times\frac{3}{39}+2\times\frac{2}{40}\times\frac{1}{39}$$

$$=\frac{11}{780}$$

(ii)(b) Let A and B be Ben's two favourite combinations.
Let $n_A\in\mathbb{Z}^+, 1\le n_A\le 5$ and $n_B\in\mathbb{Z}^+, 1\le n_B\le 5$ be the number of A and number of B respectively.

$$\frac{n_A}{40}\times\frac{n_B}{39}\times 2=\frac{1}{39}$$

$n_A n_B = 20$

$\therefore$ the possible cases given that $1\le n_A, n_B\le 5$ are:

$n_A = 5, n_B = 4$

$n_A = 4, n_B = 5$

Thus, possible A and B are:
Yellow Triangle, Green Star
Yellow Triangle, Red Square
Green Triangle, Red Square
Green Triangle, Green Star

16

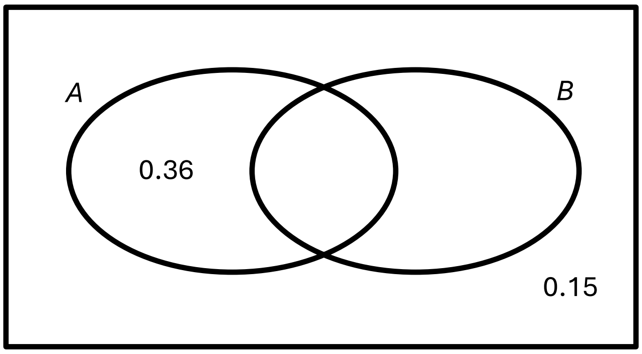
(a) $P(B') = 0.51$

$P(A' \cap B') = 0.15$

$P(A \cap B') = 0.51 - 0.15$

$= 0.36$

(b)



$P(B \mid A) = \frac{11}{29} \Rightarrow \frac{P(B \cap A)}{P(A)} = \frac{11}{29}$

$\frac{P(A) - P\left(A \cap B'\right)}{P(A)} = \frac{11}{29}$

$\frac{P(A) - 0.36}{P(A)} = \frac{11}{29}$

$29P(A) - 10.44 = 11P(A)$

$18P(A) = 10.44$

$P(A) = 0.58$

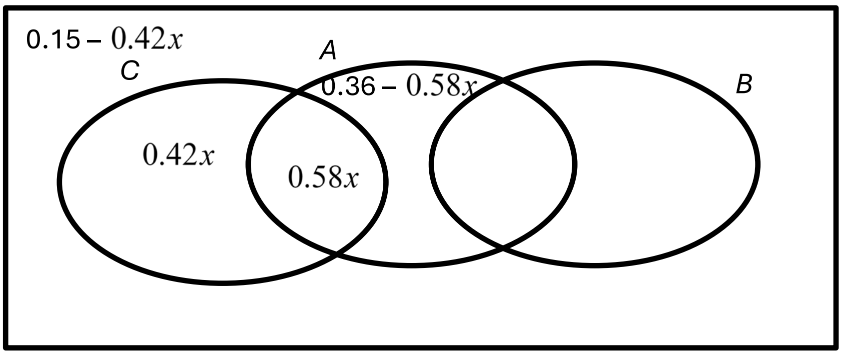
(c) Note that $P(B') = 0.51$.

Let $P\left(C\right) = x$.

Since A and C are independent,

$P\left(A \cap C\right) = 0.58x$.

Note that B and C are mutually exclusive.



$0.15 - 0.42x \geq 0$ and $0.36 - 0.58x \geq 0$

$x \leq \frac{5}{14}$ and $x \leq \frac{18}{29}$

Therefore, maximum $P\left(C\right) = \frac{5}{14}$

17

(a)(i)

B be the event that a person supports Party B;
C be the event that a person supports Party C:
V be the event that a person has voted.

Given:

$$\mathrm{P}(V|A) = 0.55, \ \ \mathrm{P}(V|B) = 0.86, \ \ \mathrm{P}(V|C) = 0.42$$

$$\begin{aligned} &\mathrm{P}(V) \\ &= \mathrm{P}(V \cap A) + \mathrm{P}(V \cap B) + \mathrm{P}(V \cap C) \\ &= \mathrm{P}(V|A)\mathrm{P}(A) + \mathrm{P}(V|B)\mathrm{P}(B) + \mathrm{P}(V|C)\mathrm{P}(C) \\ &= (0.55)(0.3) + (0.86)(0.5) + (0.42)(0.2) \\ &= 0.679 \end{aligned}$$

(a)(ii)

$$\mathrm{P}(B|V) = \frac{\mathrm{P}(V|B)\mathrm{P}(B)}{\mathrm{P}(V)} = \frac{(0.86)(0.5)}{0.679} = 0.633$$

(b)

$$\begin{aligned} &\mathrm{P}(\text{exactly one person voted}) \\ &= (0.679)(1-0.679) + (1-0.679)(0.679) \\ &= 0.435918 \end{aligned}$$

18

(a) $P(A \cap B) = 1 - P(A' \cup B')$

$= 1 - 0.82$

$= 0.18$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$0.4 = \frac{0.18}{P(B)}$$

$P(B) = 0.45$

(b) $P(B') = 1 - P(B)$

$= 1 - 0.45$

$= 0.55$

$A \cap B' \subseteq B'$

$P(A \cap B') \leq 0.55$

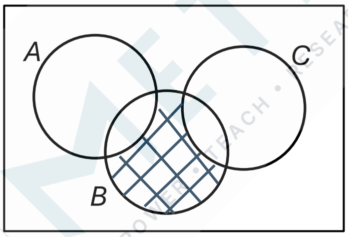
(c) B & C independent.

Hence $P(B \cap C) = P(B)P(C)$

$= 0.45 \times 0.1$

$= 0.045$

A & C are mutually exclusive.



Hence $P(A' \cap B \cap C')$

$= P(B) - P(A \cap B) - P(B \cap C)$

$= 0.45 - 0.18 - 0.045$

$= 0.225$