H2 MATHEMATICS (9758)

TOPIC ***SYSTEM OF LINEAR EQUATIONS***

1

$\mathrm{f}(x) = ax^3 + bx + c$

At $(1,10)$,

$10 = a(1)^3 + b(1) + c$

$\Rightarrow \quad a + b + c = 10$

At $(-2,12)$,

$12 = a(-2)^3 + b(-2) + c$

$\Rightarrow \quad -8a - 2b + c = 12$

Since that $\mathrm{f}(x)$ is divisible by $x-2$, $\mathrm{f}(2) = 0$.

$0 = a(2)^3 + b(2) + c$

$\Rightarrow \quad 8a + 2b + c = 0$

Using GC,

$a = -\dfrac{7}{3}, \quad b = \dfrac{19}{3}, \quad c = 6$

2

Sub $x = 0$, $A + B + C = 2$

$\dfrac{\mathrm{d}y}{\mathrm{d}x} = A\mathrm{e}^{x} + 2B\mathrm{e}^{2x} - 2C\mathrm{e}^{-2x}$

Sub $x = 0$, $A + 2B - 2C = -3$

$\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = A\mathrm{e}^{x} + 4B\mathrm{e}^{2x} + 4C\mathrm{e}^{-2x}$

Sub $x = 0$, $A + 4B + 4C = 11$

Solving simultaneously, $A = -1, B = 1, C = 2$.

So particular solution is $y = \mathrm{f}(x) = -\mathrm{e}^{x} + \mathrm{e}^{2x} + 2\mathrm{e}^{-2x}$.

3

$\mathrm{f}(x)=ax^3+bx^2+cx+d$

$\mathrm{f}(-1)=-a+b-c+d=-15 \quad -(1)$

$\mathrm{f}(2)=8a+4b+2c+d=3 \quad -(2)$

$\mathrm{f}'(x)=3ax^2+2bx+c$

$\mathrm{f}'(1)=3a+2b+c=0 \quad -(3)$

$\int_0^2 \mathrm{f}(x)\ \mathrm{d}x=6 \Rightarrow \int_0^2 (ax^3+bx^2+cx+d)\ \mathrm{d}x=5$

$\left[\frac{1}{4}ax^4+\frac{1}{3}bx^3+\frac{1}{2}cx^2+dx\right]_0^2=5$

$4a+\frac{8}{3}b+2c+2d=5 \quad -(4)$

Using GC to solve (1), (2), (3), (4)

$a=1.5,\ b=-6,\ c=7.5,\ d=0$

4

(i) $u_n=2u_{n-1}-15n^2+60n+A$

$u_2=2u_1-15(2)^2+60(2)+A$

$4=2(2)-60+120+A \Rightarrow A=-60$

$u_3=2u_2-15(3)^2+60(3)-60=2(4)-135+180-60=-7$

(ii) Given $u_1=2$, thus $2=4p-q+r$ ----Equation (1)

Given $u_2=4$, thus $4=8p-4q+r$ ---Equation (2)

Given $u_3=-7$, thus $-7=16p-9q+r$ ---Equation (3)

Solving, $p=-\frac{43}{4},\ q=-15,\ r=30$

5

$\mathrm{f}(x)=x^3+ax^2+bx+c$

$-32=1+a+b+c$

$\mathrm{f}'(x)=3x^2+2ax+b$

$0=3+2a+b$

$y=\frac{1}{\mathrm{f}(x)}$ has a vertical asymptote at $x=5$ implies that $y=\mathrm{f}(x)$ has an x-intercept at $x=5$.

$0=125+25a+5b+c$

$$\begin{cases} a+b+c=-33 \\ 2a+b=-3 \\ 25a+5b+c=-125 \end{cases}$$

By GC, $\mathrm{f}(x)=x^3-5x^2+7x-35$.

[$a=-5, b=7, c=-35$]

6

(a)

respectively.

$x + y + z = 38$ …Eq(1)

$3x + y + 0z = 54$ …Eq(2)

$x + y - 2z = 8$ …Eq(3)

From GC, $x = 13$, $y = 15$ and $z = 10$

$\therefore$ Lucy's favorite team won 13 games this season.

(b) Points scored by Mark's favorite team $= 3(13 - 2) + (15 + 5)$

$= 53 < 54$

Hence Lucy's favorite team performed better this season.

Alternatively,

Since Mark's favorite team won 2 games fewer and drew 5 games more,
this team scored $-3(2) + 5(1) = -1$ point more.

Hence Lucy's favorite team performed better this season.

7

Let \$$x$, \$$y$ and \$$z$ denote the usual selling price of a small, medium and large bag of Griffles popcorn respectively.

To receive a total of 3 small, 7 medium and 1 large bag of Griffles popcorn, Beatrice bought 2 small, 5 medium and 1 large bag of popcorn.

$0.95(3x + 7y + z) = 85.5 \quad \Rightarrow 3x + 7y + z = 90 \cdots (1)$

$2x + 5y + z = 85.5 - 18.50 \quad \Rightarrow 2x + 5y + z = 67 \cdots (2)$

$z = 2.4x \quad \Rightarrow 2.4x - z = 0 \cdots (3)$

On solving, $x = 5$, $y = 9$ and $z = 12$

The usual selling price of a small, medium and large bag of Griffles popcorn is \$5, \$9 and \$12 respectively.