1

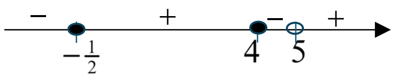
(i) $\dfrac{x^2-14}{x-5} \geq 2-x$

$\dfrac{x^2-14-(2-x)(x-5)}{x-5} \geq 0$

$\dfrac{x^2-14-\left(2x-10-x^2+5x\right)}{x-5} \geq 0$

$\dfrac{2x^2-7x-4}{x-5} \geq 0$

$\dfrac{2x+1)(x-4)}{x-5} \geq 0$



$-\dfrac{1}{2} \leq x \leq 4 \quad \text{or} \quad x > 5$

(ii) To solve $\dfrac{x^2-14}{|x|-5} \geq 2-|x|$, replace x with $|x|$

$-\dfrac{1}{2} \leq |x| \leq 4 \quad \text{or} \quad |x| > 5$

$\Rightarrow 0 \leq |x| \leq 4 \quad \text{or} \quad |x| > 5$

$-4 \leq x \leq 4 \quad \text{or} \quad x < -5 \quad \text{or} \quad x > 5$

2

$$\frac{x\ \ +2x-5}{x^2-2x}<2$$

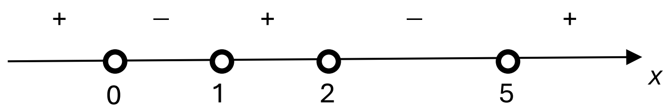
$$\frac{x^2+2x-5}{x^2-2x}-2<0$$

$$\frac{x^2+2x-5-2x^2+4x}{x^2-2x}<0$$

$$\frac{-x^2+6x-5}{x(x-2)}<0$$

$$\frac{x^2-6x+5}{x(x-2)}>0$$

$$\frac{(x-1)(x-5)}{x(x-2)}>0$$

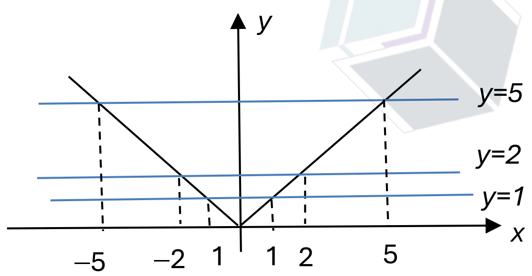


$\therefore\ \ x<0$ or $1<x<2$ or $x>5$.

Solution of $\frac{x^2+2x-5}{x^2-2x}>2$ is the complementary of solution of $\frac{x^2+2x-5}{x^2-2x}<2$

So, replacing x with $|x|$, solution of $\frac{x^2+2|x|-5}{x^2-2|x|}>2$ will be

$$0<|x|<1 \quad \text{or} \quad 2<|x|<5$$



For $0<|x|<1$, $-1<x<1,\ x\neq 0$

For $2<|x|<5$, $-5<x<-2$ or $2<x<5$

Thus, range of values: $-5<x<-2$ or $2<x<5$ or $-1<x<1,\ x\neq 0$

3

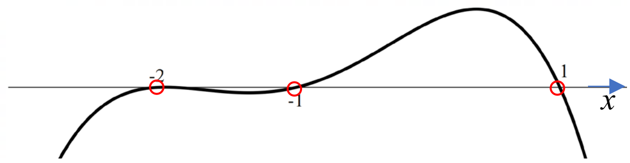
$$\frac{9}{(1-x)(1+x)} < \frac{x+5}{x+1}, \quad x \neq \pm 1$$

$$\frac{9-(x+5)(1-x)}{(1-x)(1+x)} < 0$$

$$\frac{x^2+4x+4}{(1-x)(1+x)} < 0$$

$$(x+2)^2(1-x)(1+x) < 0 \cdots (*)$$

<u>Method 1:</u>



$\therefore x < -2 \quad \text{or} \quad -2 < x < -1 \quad \text{or} \quad x > 1$

<u>Method 2:</u>

Since $(x+2)^2 \geq 0$ for $x \in \mathbb{R}$, (*) is equivalent to

$(1-x)(1+x) < 0$ and $x \neq -2$

$\therefore x < -1 \quad \text{or} \quad x > 1 \quad \text{and} \quad x \neq -2$

4

(i) $\dfrac{4x^2-4x+1}{1+x-2x^2}<0$

$$\frac{(2x-1)^2}{(1-x)(2x+1)}<0$$

Method 1:

$$\frac{(2x-1)^2}{(1-x)(2x+1)}<0$$

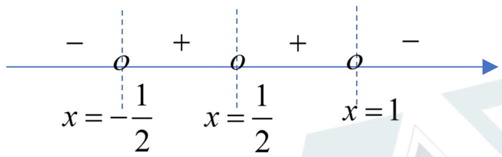
Since $(2x-1)^2 \geq 0$ for all real values of x,

$(1-x)(2x+1)<0$

$\therefore x<-\dfrac{1}{2}$ or $x>1$

Method 2:

$$\frac{(2x-1)^2}{(1-x)(2x+1)}<0$$



$\therefore x<-\dfrac{1}{2}$ or $x>1$

(ii) $\dfrac{(2^{x+1}-1)^2}{1+2^x-2^{2x+1}}\leq 0$

$$\frac{(2(2^x)-1)^2}{1+(2^x)-2(2^x)^2}\leq 0$$

Replace x with 2^x,

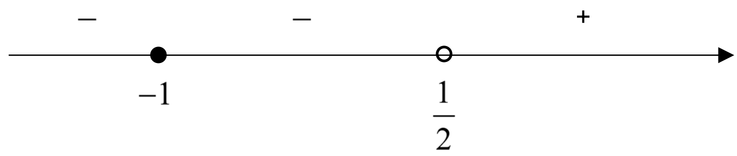
$2^x<-\dfrac{1}{2}$ OR $2^x>1$ OR $2^x=\dfrac{1}{2}$

(rejected, since 2^x is positive for all real values of x.) | $x>0$ | $x=-1$

5

(a) $\dfrac{x\ \ +2x+1}{2x-1}\geq 0$

$\dfrac{(x+1)^2}{2x-1}\geq 0$



Using test point method, $x=-1$ or $x>\frac{1}{2}$.

(b) Let $x=\sin\theta$.

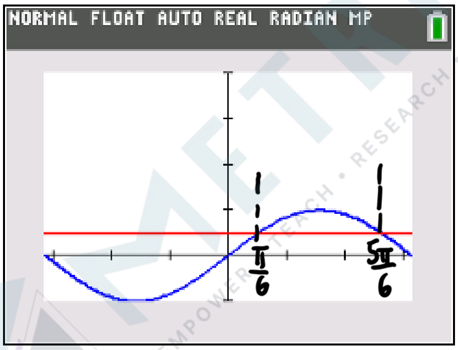
From (a), $\sin\theta=-1$ or $\sin\theta>\frac{1}{2}$

For $\sin\theta=-1$

$\theta=-\frac{\pi}{2}$

For $\sin\theta>\frac{1}{2}$

$\frac{\pi}{6}<\theta<\frac{5\pi}{6}$



6

(a)

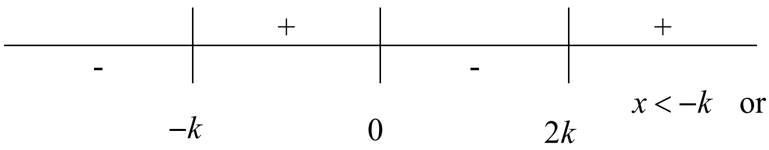
$$\frac{2x^2-2k^2}{x}<x+k$$

$$\frac{2\left(x^2-k^2\right)}{x}-\left(x+k\right)<0$$

$$\frac{2\left(x-k\right)\left(x+k\right)-x\left(x+k\right)}{x}<0$$

$$\frac{\left(2x-2k-x\right)\left(x+k\right)}{x}<0$$

$$\frac{\left(x-2k\right)\left(x+k\right)}{x}<0$$



$x<-k$ or $0<x<2k$

(b) Let $k=\sqrt{5}$ and replacing x with $|x|$.

$$\frac{2|x|^2-10}{|x|}<|x|+\sqrt{5}$$

$$\frac{2x^2-10}{|x|}<|x|+\sqrt{5} \qquad \text{since } |x|^2=x^2$$

Hence,

$|x|<-\sqrt{5}$ (rejected $\because |x|\geq 0$)

or $0<|x|<2\sqrt{5}$

$-2\sqrt{5}<x<0$ or $0<x<2\sqrt{5}$

(Also, $-2\sqrt{5}<x<2\sqrt{5}$ and $x\neq 0$)

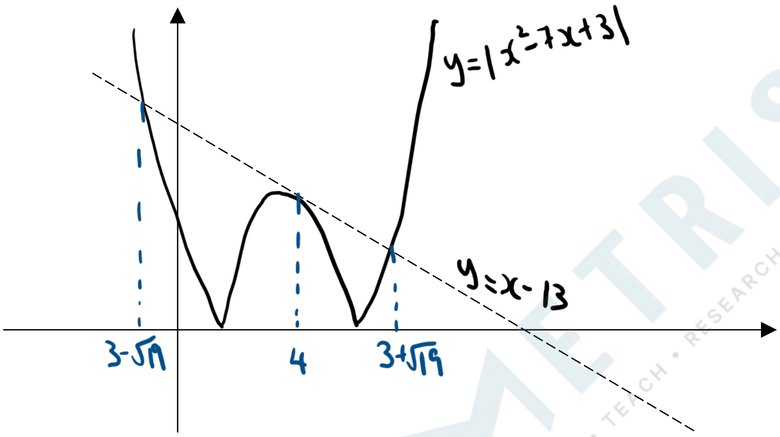
7

(a) Consider the following cases:

$x^2 - 7x + 3 = 13 - x$ $x^2 - 6x - 10 = 0$ $x = \dfrac{6 \pm \sqrt{6^2 - 4(1)(-10)}}{2(1)}$ $= \dfrac{6 \pm \sqrt{76}}{2} = \dfrac{6 \pm 2\sqrt{19}}{2}$ $= 3 \pm \sqrt{19}$	$-\left(x^2 - 7x + 3\right) = 13 - x$ $x^2 - 8x + 16 = 0$ $x = \dfrac{8 \pm \sqrt{8^2 - 4(1)(16)}}{2(1)}$ $= 4$

The roots are $3 \pm \sqrt{19}$ and 4.

(b)



From the graph, the solution is $3 - \sqrt{19} < x < 4$ or $4 < x < 3 + \sqrt{19}$

8

(a) $\left|3x^2+8x-3\right|=3-x$ ------ (*)

$3x^2+8x-3=3-x$ or $-\left(3x^2+8x-3\right)=3-x$

$3x^2+9x-6=0$ or $3x^2+7x=0$

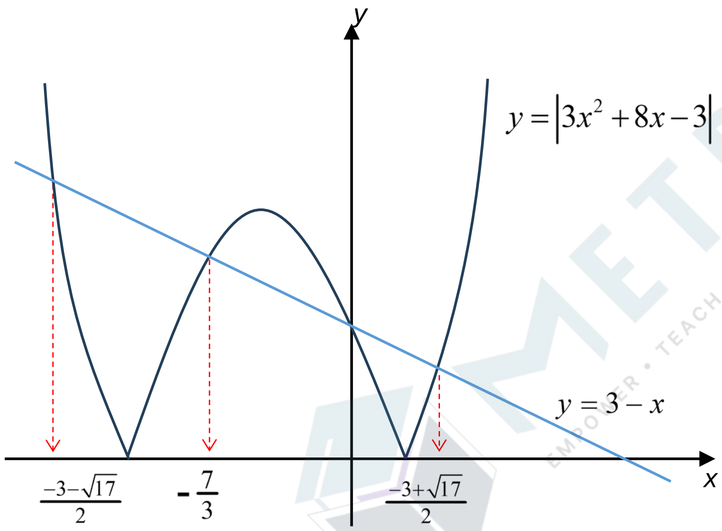
$x^2+3x-2=0$ or $x(3x+7)=0$

$x=\dfrac{-3\pm\sqrt{9-4(1)(-2)}}{2}$ $\qquad x=0$ or $-\dfrac{7}{3}$

$=\dfrac{-3\pm\sqrt{17}}{2}$

(Alternative method: Squaring both sides and so on)

(b)



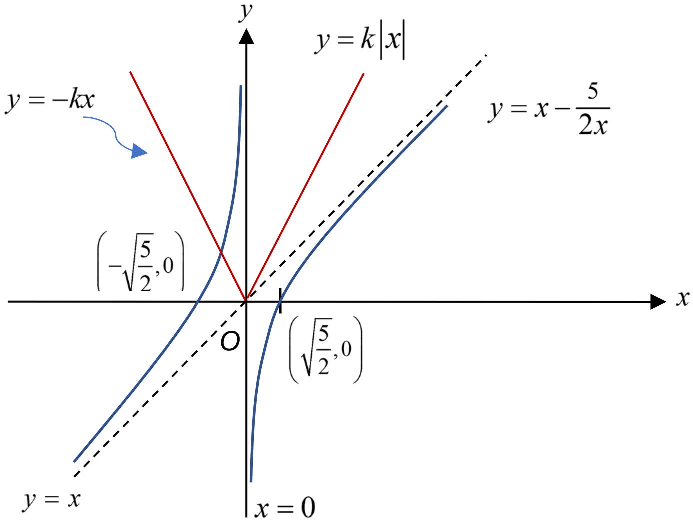
As seen from the graphs, for

$\left|3x^2+8x-3\right|\geq 3-x$

$x\leq\dfrac{-3-\sqrt{17}}{2}$ or $-\dfrac{7}{3}\leq x\leq 0$ or $x\geq\dfrac{-3+\sqrt{17}}{2}$

9

(a)



(b) For point of intersection between the 2 graphs,

$$x - \frac{5}{2x} = -kx$$

$$2x^2 - 5 = -2kx^2$$

$$2x^2(k+1) = 5$$

$$x = \pm\sqrt{\frac{5}{2(k+1)}}$$

Since $x < 0$, $x = -\sqrt{\frac{5}{2(k+1)}}$.

Hence, for $x - \frac{5}{2x} \le k|x|$,

$x \le -\sqrt{\frac{5}{2(k+1)}}$ or $x > 0$.

(c) Replace x by $-x$ and $k = 3$

$x \ge \sqrt{\frac{5}{2(3+1)}}$ or $x < 0$

$x \ge \sqrt{\frac{5}{8}}$ or $x < 0$